Mas314 Matlab Lab III

The goal of this lab is to study the convergence behavior of the following numerical methods:

- `Secant method and method of false position;`
- `Aitken's` Δ^2 `method and Steffensen's method;`
- `Horner's method and Müller's method.`

Read the description of each method (cf. Sections 2.5 and 2.6 of the textbook) and do the numerical tests.

Task I. The **method of false position** generates the sequence $\{p_n\}$ in the same manner as the **secant method**, but includes a test for root bracketing as the **bisection method**.

Solve the following problem and use `matlab` to graph the function $f(x)$ and coordinate sequence $\{(p_n, 0)\}$.

The polynomial

$$f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$$

has two real zeros, one in $[-1, 0]$ *and the other in* $[0, 1]$. *Attempt to approximate these zeros within* 10^{-6} *using the*

(a) *Method of false position;*

(b) *Secant method;*

(c) *Newton's method.*

Use the endpoints of each interval as the initial approximation in (a) and (b), and the midpoint as the initial approximation in (c).

Answer for reference:

(a) For $p_0 = -1$ and $p_1 = 0$, we have $p_{17} = -0.04065850$, and for $p_0 = 0$ and $p_1 = 1$, we have $p_9 = 0.9623984$;

(b) For $p_0 = -1$ and $p_1 = 0$, we have $p_5 = -0.04065929$, and for $p_0 = 0$ and $p_1 = 1$, we have $p_{12} = -0.04065929$;

(c) For $p_0 = -0.5$, we have $p_5 = -0.04065929$, and for $p_0 = 0.5$ we have $p_{21} = 0.9623989$.

Another test to show that the `method of false position` converges, while the `secant method` may not.

The function $f(x) = \tan(\pi x) - 6$ *has a zero at* $p = (1/\pi)\text{arctan}6 \approx 0.447431543$. *Let* $p_0 = 0$ *and* $p_1 = 0.48$ *and use ten iterations of each of the following methods to* *approximate the roots, which one is most successful and use* `matlab graph` *to explain why?*

(a) *Method of false position.*

(b) *Secant method.*

Answer for reference:

(a) *The method of false position yields that* $p_{10} = 0.442067$.

(b) *The secant method yields that* $p_{10} = -195.8950$.

Task II. `Aitken's` Δ^2 `method` can used to accelerate the convergence of a sequence that is linearly convergent. Suppose that $\{p_n\}$ is a linearly convergent sequence with limit p. We construct a new sequence $\{\hat{p}_n\}$ via

$$\hat{p}_n = p_n - \frac{(\Delta p_n)^2}{\Delta^2 p_n} = p_n - \frac{(p_{n+1} - p_n)^2}{p_{n+2} - 2p_{n+1} + p_n} := \{\Delta^2\}(p_n), \quad \text{for} \quad n \geq 0.$$

This new sequence is expected to be convergent faster than the original sequence. Verify this by considering:

The following sequences converges to 0. *Use* `Aitken's` Δ^2 `method` *to generate* $\{\hat{p}_n\}$ *until* $|\hat{p}_n| \leq 5^{-5}$, *and demonstrate the convergence rate of two sequences:*

$$(a) \;\; p_n = \frac{1}{n+1}, \qquad (b) \;\; p_n = \frac{1}{(n+1)^2}.$$

`The Steffensen's method` generates the new sequence $\{\hat{p}_n\}$ in a way different from the `Aitken's` Δ^2 `method`, refer to Page 85 for details. Use `The Steffensen's method` to solve $x^3 + 4x^2 - 10 = 0$ via the fixed-point iteration with

$$g(x) = \Big(\frac{10}{x+4}\Big)^{1/2}.$$

Show that this method converges quadratically.

Task III. `Horner's method` provides an efficient way to evaluate a polynomial $P_n(x_0)$ and its first-order derivative $P_n'(x_0)$. Use this method to compute $P_3(1)$ and $P_3'(1)$ with $P_3(x) = -x^3 - 2x^2 + 10$.

`The Müller's method` is an alternative to the `secant method` and `Newton's method` for root-finding problem. Read and program `Algorithm 2.8` on Page 93. Use this method for solving

$f(x) = 16x^4 - 40x^3 + 5x^2 + 20x + 6 = 0$ *with* $TOL = 10^{-5}$ *and* $p_0 = 0.5,\ p_1 = 1.0,\ p_2 = 1.5$.