Mas314 Matlab Lab II

The goals of this lab are

1. To review `Matlab` programming:
   - use `help` to learn the commands: `format`; `input`; `fprintf`; `sprintf`; `load`
   - Matlab Graphics: `plot, semilogy; title, xlabel, ylabel, gca, text, meshgrid; mesh; surf;`

2. A comparison of the convergence rate of the `Bisection Method, Fixed-Point Iteration` and `Newton Method`.

Suppose that $\{p_n\}_{n=0}^{\infty}$ is a sequence generated by an iteration method, and assume that $p_n \to p$ as $n \to \infty$. If

$$\lim_{n \to \infty} \frac{|p_{n+1} - p|}{|p_n - p|} = \lambda < 1,$$

then we say $\{p_n\}_{n=0}^{\infty}$ converges to p *linearly.* On the other hand, if

$$\lim_{n \to \infty} \frac{|p_{n+1} - p|}{|p_n - p|^2} = \text{Constant},$$

then we say $\{p_n\}_{n=0}^{\infty}$ converges to p *quadratically.*

Problem I: Consider the equation $x^2 - 2 = 0$, and rewrite it as $x = \frac{x}{2} + \frac{1}{2}$. Construct the iteration scheme:

$$x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}, \quad n \geq 0.$$

Use `Matlab` to show that for any $x_0 \geq 1$, we have $x_n \to \sqrt{2}$, and the convergence rate is *quadratical.*

Problem II: Consider the equation $f(x) = e^x - x - 1 = 0$. Show that the sequence

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad x_0 = 0.1,$$

converges to 0 only *linearly.* How about the following sequences?

$$\text{(i)} \quad x_{n+1} = x_n - 2\frac{f(x_n)}{f'(x_n)}, \quad x_0 = 0.1,$$

and

$$\text{(ii)} \quad x_{n+1} = x_n - \frac{f(x_n)f'(x_n)}{[f'(x_n)]^2 - f(x_n)f''(x_n)}, \quad x_0 = 0.1.$$

Problem III: Use *Matlab* to show that

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}} = \frac{1 + \sqrt{5}}{2}.$$

Explore the convergence rate.