MAS437 — Algebraic Topology

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1 Textbook

Allen Hatcher *Algebraic Topology* — available online on the author’s personal webpage. Our course covers Chapter 0 (revision of basic concepts), Chapter 1 (fundamental group), Chapter 2 (homology groups).

2 Marking scheme

30% Eight assignments. Each assignment is a problem of your choice from the relevant chapter of the textbook.

70% Final exam.

3 Plan for the semester

**Week 1** Cell complexes, homotopy equivalence. Hatcher 0.

**Week 2** Homotopy equivalence. (Chinese New Year) Hatcher 0.

**Week 3** Fundamental group, induced homomorphisms. Hatcher 1.1.

**Week 4** Van Kampen’s theorem. Hatcher 1.2.

**Week 5** Covering spaces. Hatcher 1.3.
Week 6 Covering spaces. Hatcher 1.3.

Week 7 Singular homology and simplicial homology. Hatcher 2.1.

Week 8 Homotopy invariance. Exact sequences. Hatcher 2.1.

Week 9 Excision. Hatcher 2.1.

Degree of a map $S^n \to S^n$. Hatcher 2.2.

Week 10 Cellular homology. Hatcher 2.2.

Week 11 Cellular homology. Mayer-Vietoris sequence. Hatcher 2.2.

Week 12 Applications of homology theory. Hatcher 2.B.

Enrichment: Morse theory and cohomology (not tested).

Week 13 Enrichment: Alexander’s polynomial (not tested). Revision.

4 Final exam

Content of the final exam and most relevant exercises from Hatcher.

50% Fundamental group

• Fundamental group. Definition and basic properties. Induced homomorphisms. Exercises: 1.1.12, 1.1.14, 1.1.16.

• Van Kampen’s theorem. Exercises: 1.2.7, 1.2.8, 1.2.12, 1.2.14, 1.2.16.

• Covering spaces. Lifting criterion, classification of covering spaces, deck transformations, normal coverings. Exercises: 1.3.9, 1.3.10, 1.3.12, 1.3.17, 1.3.25, 1.3.31.
50% Homology groups

• Explicit computation (singular and cellular). Homology groups of surfaces, spheres, projective spaces. Exercises: 2.1.4, 2.1.5, 2.2.9, 2.2.10, 2.2.11.

• Exact sequences. Group-theoretical meaning of short exact sequences, zeroes in exact sequences, split exact sequences, five-lemma. Long exact sequence of pair and a triple and Mayer-Vietoris sequence. Exercises: 2.1.14, 2.1.15, 2.1.16, 2.1.17, 2.1.18, 2.2.28, 2.2.29.

• Applications of homology theory. Induced homomorphisms in homology and degree of map. Exercises: 2.2.2, 2.2.6, 2.2.35.