

# New Constant-Weight Codes From Propagation Rules

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**Abstract**—This paper proposes some simple propagation rules which give rise to new binary constant-weight codes.

**Index Terms**—Constant-weight codes, cosets,  $q$ -ary codes.

## I. INTRODUCTION

THE ring  $\mathbb{Z}/q\mathbb{Z}$  is denoted  $\mathbb{Z}_q$ . We endow  $\mathbb{Z}_q^n$  with the Hamming distance metric  $\Delta$ : for  $u, v \in \mathbb{Z}_q^n$ ,  $\Delta(u, v)$  is the number of positions where  $u$  and  $v$  differ. A ( $q$ -ary) code of length  $n$  is a subset  $\mathcal{C} \subseteq \mathbb{Z}_q^n$ . The elements of  $\mathcal{C}$  are called *codewords*, and the *size* of  $\mathcal{C}$  is the number of codewords it contains. The *minimum distance* of a code  $\mathcal{C}$  is  $\Delta(\mathcal{C}) = \min_{u, v \in \mathcal{C}, u \neq v} \Delta(u, v)$ . We often denote by  $(n, d)_q$ -code a  $q$ -ary code of length  $n$  and minimum distance at least  $d$ .

The *weight*,  $\text{wt}(u)$ , of  $u \in \mathbb{Z}_q^n$  is its distance from the origin, that is,  $\text{wt}(u) = \Delta(u, 0)$ . For  $0 \leq w \leq n$ , the ( $q$ -ary) *Johnson space*  $J_q^n(w)$  is the set of all elements of  $\mathbb{Z}_q^n$  having weight  $w$ , that is,  $J_q^n(w) = \{u \in \mathbb{Z}_q^n : \text{wt}(u) = w\}$ . A ( $q$ -ary) *constant-weight code* of length  $n$ , distance  $d$ , and weight  $w$ , denoted  $(n, d, w)_q$ -code, is a code  $\mathcal{C} \subseteq J_q^n(w)$  such that  $\Delta(\mathcal{C}) \geq d$ .

We adopt the convention throughout this paper that if  $q$  is not specified, then we assume  $q = 2$ . Hence, for example, an  $(n, d, w)$ -code refers to an  $(n, d, w)_2$ -code, and  $J^n(w)$  refers to  $J_2^n(w)$ .

Binary constant-weight codes have been extensively studied for more than four decades due to their fascinating combinatorial structures and applications [1]–[19]. Given  $n, d$ , and  $w$ , the central problem of interest in binary constant-weight codes is in the determination of  $A(n, d, w)$ , the largest possible size of an  $(n, d, w)$ -code. Exact values of  $A(n, d, w)$  are known only for a few infinite families of parameters  $n, d$ , and  $w$ , and in some other sporadic instances (see, for example, [3], [4]). In light of the difficulty of determining  $A(n, d, w)$  exactly, various bounds have also been developed. There are two online tables devoted to bounds on  $A(n, d, w)$ : one maintained by Rain and Sloane [20] and the other by Smith and Montemanni [21]. While the former table considers codes of lengths not exceeding 63, the latter table focuses mainly on codes for lengths between 29 and 63, having small weights.

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In this paper, we present simple propagation rules for binary constant-weight codes through  $q$ -ary codes. It turns out that some good binary constant-weight codes can be obtained from these propagation rules. In particular, we improve on a number of bounds in the online tables of Rain and Sloane [20], and Smith and Montemanni [21].

We remark that the table of Smith and Montemanni [21] was created because the table of Rains and Sloane [20] had not been updated for many years. For code parameters that are not covered by Smith and Montemanni [21], we have checked against recent literature, to the best of our efforts, in ascertaining that our results here do indeed improve upon existing results.

## II. PROPAGATION RULES

In this section, we present some simple propagation rules for binary constant-weight codes from  $q$ -ary codes. We begin with a simple observation.

Let  $\mathcal{C} \subseteq \mathbb{Z}_q^n$ . For  $u \in \mathbb{Z}_q^n$ , we denote by  $u + \mathcal{C}$  the *coset* of  $\mathcal{C}$

$$\{u + c : c \in \mathcal{C}\}.$$

We also embed  $\mathbb{Z}_2$  into  $\mathbb{Z}_q$ . It is evident that  $(u + \mathcal{C}) \cap J^n(w)$  is a binary constant-weight code of weight  $w$  and size  $N = |(u + \mathcal{C}) \cap J^n(w)|$ . Since the minimum distance  $d'$  of  $(u + \mathcal{C}) \cap J^n(w)$  is at least  $d$  and  $d'$  must be even, it follows that  $d' \geq 2\lfloor(d+1)/2\rfloor$ . Thus, we have the following.

**Theorem 2.1:** Let  $0 < w < n$ . If there exists an  $(n, d)_q$ -code  $\mathcal{C}$ , then there exists an  $(n, 2\lfloor(d+1)/2\rfloor, w)$ -code of size  $N$ , where

$$N = \max_{u \in \mathbb{Z}_q^n} |(u + \mathcal{C}) \cap J^n(w)|.$$

A simple bound on the size of the constant-weight codes in Theorem 2.1 can be obtained by considering the average size of the cosets.

**Theorem 2.2:** Let  $0 < w < n$ . If there exists an  $(n, d)_q$ -code of size  $M$ , then

$$A(n, 2\lfloor(d+1)/2\rfloor, w) \geq \left\lceil \frac{M \binom{n}{w}}{q^n} \right\rceil.$$

*Proof:* Let  $\mathcal{C}$  be an  $(n, d)_q$ -code of size  $M$ . Let  $u_1, u_2, \dots, u_{q^n}$  denote all the elements of  $\mathbb{Z}_q^n$ , and let  $v_1, v_2, \dots, v_{\binom{n}{w}}$  denote all the elements of  $J^n(w)$ . Define

$$\delta_{i,j} = \begin{cases} 1, & \text{if } v_j \in u_i + \mathcal{C} \\ 0, & \text{if } v_j \notin u_i + \mathcal{C}. \end{cases}$$

For each  $v_j \in J^n(w)$ , there are  $M$  elements  $u_i \in \mathbb{Z}_q^n$  such that  $u_i + \mathcal{C}$  contains  $v_j$  (to see this, note that  $v_j \in u_i + \mathcal{C}$  if and only if  $u_i = v_j + c$  for some  $c \in \mathcal{C}$ ). Thus

$$\sum_{1 \leq i \leq q^n} \sum_{1 \leq j \leq \binom{n}{w}} \delta_{i,j} = M \binom{n}{w}.$$

Hence, there exists at least one  $\ell, 1 \leq \ell \leq q^n$ , such that

$$\sum_{1 \leq j \leq \binom{n}{w}} \delta_{\ell,j} \geq \frac{M \binom{n}{w}}{q^n}.$$

The theorem now follows by noting that the size of  $(u_\ell + \mathcal{C}) \cap J^n(w)$  is precisely  $\sum_{1 \leq j \leq \binom{n}{w}} \delta_{\ell,j}$ , and we have seen above that  $(u_\ell + \mathcal{C}) \cap J^n(w)$  is an  $(n, 2\lfloor(d+1)/2\rfloor, w)$ -code.  $\square$

Next, we consider binary constant-weight codes of length  $n+1$  from  $q$ -ary codes of length  $n$ .

*Theorem 2.3:* Let  $0 < w < n$ . Suppose there exists an  $(n, d)_q$ -code  $\mathcal{C}$  of size  $M$ . Then,

- i) there exists an  $(n+1, 2\lfloor(d+1)/2\rfloor, w)$ -code of size  $N$ , where

$$N = \max_{u \in \mathbb{Z}_q^n} |(u + \mathcal{C}) \cap (J^n(w-1) \cup J^n(w))|;$$

- ii)

$$A(n+1, 2\lfloor(d+1)/2\rfloor, w) \geq \left\lceil \frac{M \left( \binom{n}{w-1} + \binom{n}{w} \right)}{q^n} \right\rceil.$$

*Proof:*

- i) Let  $u \in \mathbb{Z}_q^n$  such that  $|(u + \mathcal{C}) \cap (J^n(w-1) \cup J^n(w))|$  achieves the maximum size  $N$ . It is clear that  $\mathcal{C}' = (u + \mathcal{C}) \cap (J^n(w-1) \cup J^n(w))$  is an  $(n, d)$ -code, where each codeword has weight either  $w-1$  or  $w$ . To each codeword  $c \in \mathcal{C}'$ , append a new coordinate which takes on value one if  $\text{wt}(c) = w-1$  and value zero if  $\text{wt}(c) = w$ . The set of resulting codewords is an  $(n+1, 2\lfloor(d+1)/2\rfloor, w)$ -code.
- ii) Using the same arguments as in the proof of Theorem 2.2, we get an  $(n, d)$ -code of size  $M(\binom{n}{w-1} + \binom{n}{w})/q^n$ , in which the weight of every codeword is either  $w-1$  or  $w$ . By appending a new coordinate to every codeword as in (i) above, we get an  $(n+1, 2\lfloor(d+1)/2\rfloor, w)$ -code of the required size.  $\square$

### III. EXAMPLES

We provide some examples where the propagation rules given by Theorems 2.2 and 2.3 lead to improved bounds on  $A(n, d, w)$ .

In the tables of this section, a bold entry indicates that the size of the code constructed here is larger than any known codes of the same parameters, and a entry superscripted by an asterisk indicates that the size of the code constructed here is of the same size as the best known code of the same parameters.  $M_{\max}$  denotes the lower bound on  $A(n, d, w)$  given by Theorems 2.1 or

TABLE I  
SOME CONSTANT-WEIGHT CODES OF DISTANCE EIGHT

Lower Bounds on $A(63, 8, w)$			Lower Bounds on $A(64, 8, w)$		
$w$	$M_{\text{avg}}$	$M_{\text{RS}}$	$w$	$M_{\text{avg}}$	$M_{\text{RS}}$
7	<b>8443</b>	7182	7	<b>9480</b>	8064
8	<b>59096</b>	50274	8	<b>67538</b>	57456
9	<b>361141</b>	-	9	<b>420236</b>	-
10	<b>1950158</b>	-	10	<b>2311298</b>	-
11	<b>9396214</b>	-	11	<b>11346372</b>	-
12	<b>40716926</b>	-	12	<b>50113140</b>	-
13	<b>159735632</b>	-	13	<b>200452558</b>	-
14	<b>570484400</b>	-	14	<b>730220032</b>	-

2.3(i), and  $M_{\text{avg}}$  denotes the lower bound on  $A(n, d, w)$  given by Theorems 2.2 or 2.3(ii).  $M_{\text{RS}}$  denotes the lower bound on  $A(n, d, w)$  in the tables of Rains and Sloane [20].

*Example 3.1:* Let  $\mathcal{C}$  be the Goethals  $(63, 7)$ -code of size  $2^{47}$  [22] (see [23, Ch. 5] for the structure of this code).

- Theorems 2.2 and 2.3(ii) give

$$A(63, 8, w) \geq \left\lceil \binom{63}{w} / 2^{16} \right\rceil,$$

$$A(64, 8, w) \geq \left\lceil \left( \binom{63}{w-1} + \binom{63}{w} \right) / 2^{16} \right\rceil.$$

The implications of these bounds are given in Table I.

- Shortening  $\mathcal{C}$  at the last  $i$  positions,  $1 \leq i \leq 46$ , results in a  $(63-i, 7)$ -code of size  $2^{47-i}$ . It follows from Theorem 2.2 that there exists a  $(63-i, 8, 7)$ -code of size  $\binom{63-i}{7} / 2^{16}$ . In particular, when  $i \in \{1, 2, 3\}$ , this implies

$$A(62, 8, 7) \geq 7505 \tag{1}$$

$$A(61, 8, 7) \geq 6657 \tag{2}$$

$$A(60, 8, 7) \geq 5894. \tag{3}$$

The three lower bounds (1)–(3) improve those in [21] (the corresponding lower bounds given there are 6693, 6223, and 5770, respectively, obtained by Smith *et al.* [13]).

*Example 3.2:* Let  $\mathcal{C}$  be the Preparata  $(63, 5)$ -code of size  $2^{52}$  [24] (see [23, Ch. 5] for the structure of this code). Theorems 2.2 and 2.3(ii) give

$$A(63, 6, w) \geq \left\lceil \binom{63}{w} / 2^{11} \right\rceil$$

$$A(64, 6, w) \geq \left\lceil \left( \binom{63}{w-1} + \binom{63}{w} \right) / 2^{11} \right\rceil.$$

We also found via computation cosets of  $\mathcal{C}$  achieving the maximum in Theorems 2.1 and 2.3(i). The results are given in Tables II and III.

*Example 3.3:* Let  $\mathcal{C}$  be the (linear)  $(31, 9)$ -code of size  $2^{13}$  constructed by Grassl [25].

- We found via computation cosets of  $\mathcal{C}$  achieving the maximum in Theorems 2.1 and 2.3(i). The results are given in Table IV.
- Shortening  $\mathcal{C}$  at the last two positions results in a (linear)  $(29, 9)$ -code of size  $2^{11}$ . We found, via computation, cosets

TABLE II  
LOWER BOUNDS ON  $A(63, 6, w)$

$w$	$M_{\text{avg}}$	$M_{\text{max}}$	$M_{\text{RS}}$
5	3433	3906*	3906
6	33177	37758*	37758
7	<b>270152</b>	<b>270468</b>	264771
8	<b>1891062</b>	<b>1893276</b>	1853397
9	11556490	11594310*	11594310
10	62405042	62609274*	62609274
11	<b>300678837</b>	<b>300700062</b>	300496392
12	<b>1302941625</b>	<b>1302990507</b>	1302151032
13	5111540218	5112164988*	5112164988
14	18255500778	18257732100*	18257732100

TABLE III  
LOWER BOUNDS ON  $A(64, 6, w)$

$w$	$M_{\text{avg}}$	$M_{\text{max}}$	$M_{\text{RS}}$
5	<b>3723</b>	<b>3906</b>	-
6	36609	41664*	41664
7	<b>303329</b>	<b>303354</b>	-
8	<b>2161214</b>	<b>2163744</b>	2118168
9	<b>13447552</b>	<b>13447707</b>	-
10	73961530	74203584*	74203584
11	<b>363083878</b>	<b>363105666</b>	-
12	<b>1603620460</b>	<b>1603680624</b>	1602647424
13	<b>6414481842</b>	<b>6414487191</b>	-
14	23367040996	23369897088*	23369897088

TABLE IV  
SOME CONSTANT-WEIGHT CODES OF DISTANCE 10

Lower Bounds on $A(31, 10, w)$			Lower Bounds on $A(32, 10, w)$		
$w$	$M_{\text{max}}$	$M_{\text{RS}}$	$w$	$M_{\text{max}}$	$M_{\text{RS}}$
11	<b>387</b>	-	11	<b>585</b>	-
12	<b>612</b>	-	12	<b>953</b>	-
13	<b>872</b>	-	13	<b>1443</b>	-
14	<b>1106</b>	-	14	<b>1923</b>	-

of this shortened code achieving the maximum in Theorem 2.3(i). This gives  $A(30, 10, 12) \geq 390$ . Lower bounds on  $A(30, 10, 12)$  are previously not known.

*Example 3.4:* Let  $\mathcal{C}$  be the (linear) BCH  $(31, 11)$ -code of size  $2^{11}$  [26], [27] (see [23, Ch. 8] for the structure of this code).

- We found, via computation, cosets of  $\mathcal{C}$  achieving the maximum in Theorems 2.1 and 2.3(i). The results are given in Table V.
- Shortening  $\mathcal{C}$  at the last  $i$  positions,  $i \in \{1, 2\}$ , results in a  $(31-i, 11)$ -code of size  $2^{11-i}$ . We found, via computation, cosets of these shortened codes achieving the maximum in Theorems 2.1 and 2.3 (i). These provide the lower bounds

$$A(29, 12, 11) \geq 76$$

$$A(29, 12, 12) \geq 114$$

$$A(29, 12, 13) \geq 140,$$

and

$$A(30, 12, 10) \geq 66,$$

$$A(30, 12, 11) \geq 120,$$

$$A(30, 12, 12) \geq 190$$

$$A(30, 12, 13) \geq 234$$

$$A(30, 12, 14) \geq 288.$$

TABLE V  
SOME CONSTANT-WEIGHT CODES OF DISTANCE 12

Lower Bounds on $A(31, 12, w)$			Lower Bounds on $A(32, 12, w)$		
$w$	$M_{\text{max}}$	$M_{\text{RS}}$	$w$	$M_{\text{max}}$	$M_{\text{RS}}$
9	<b>40</b>	-	9	<b>40</b>	-
10	<b>87</b>	-	10	<b>122</b>	-
11	<b>186</b>	-	11	<b>186</b>	-
12	<b>310</b>	-	12	<b>496</b>	-
13	<b>400</b>	-	13	<b>400</b>	-
14	<b>510</b>	-	14	<b>900</b>	-

Previously, no lower bounds are known on  $A(n, 12, w)$  for these parameter sets.

*Example 3.5:* Let  $\mathcal{C}$  be the (linear)  $(31, 13)$ -code of size  $2^7$  constructed by Grassl [25]. We found, via computation, cosets of  $\mathcal{C}$  achieving the maximum in Theorem 2.3(i). These provide the lower bounds

$$A(32, 14, 12) \geq 29$$

$$A(32, 14, 13) \geq 42.$$

Lower bounds on  $A(32, 14, w)$ ,  $w \in \{12, 13\}$ , are previously not known.

*Example 3.6:* Let  $\mathcal{C}_0$  be the (linear) Reed–Muller  $(32, 16)$ -code of size  $2^6$ , and let  $\mathcal{C}$  be the code obtained from  $\mathcal{C}_0$  by puncturing it at the last position. Then  $\mathcal{C}$  is a  $(31, 15)$ -code of size  $2^6$ . We found, via computation, cosets of  $\mathcal{C}$  achieving the maximum in Theorems 2.1 and 2.3(i). These provide the lower bounds

$$A(n, 16, 13) \geq 16$$

$$A(n, 16, 14) \geq 21$$

$$A(n, 16, 15) \geq 31$$

for  $n \in \{31, 32\}$ . Lower bounds on  $A(n, 16, w)$  are previously not known for these parameters.

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