A Few More Large Sets of t-Designs

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Abstract: We construct several new large sets of t-designs that are invariant under Frobenius groups, and discuss their consequences. These large sets give rise to further new large sets by means of known recursive constructions including an infinite family of large sets of $3-(v, 4, \lambda)$ designs. © 1998 John Wiley & Sons, Inc. J Combin Designs 6: 293–308, 1998

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1. INTRODUCTION

The study of large sets of t-designs constitutes an important part of combinatorial design theory. Teirlink's remarkable proof of the existence of t-designs for all t involves constructing large sets of t-designs [37, 38]. Large sets of t-designs also have applications in cryptography [33]. Relatively recent work in the construction of large sets of t-designs and related structures includes [1, 2, 6, 8, 14, 17, 19, 20, 25, 34, 41]. A survey on the existence of large sets of t-designs can be found in [23].

If X is a finite set, $\mathcal{S}_X$ denotes the symmetric group on the symbols of X, $\binom{X}{k}$ the collection of all k-subsets of X, and 2^X the power set of X.

If $\gamma \in \mathcal{S}_X, x \in X, B \in \binom{X}{k}, \mathcal{B} \subseteq 2^X$, and $\mathbb{B} \subseteq 2^{2^X}$, we denote by $x^\gamma, B^\gamma, \mathcal{B}^\gamma$, and $\mathbb{B}^\gamma$ the images under γ of $x, B, \mathcal{B}$, and $\mathbb{B}$, respectively.

A *t-design*, or more specifically a t-(v, k, λ) *design*, is a pair $(X, \mathcal{B})$, where $|X| = v$ and $\mathcal{B} \subseteq \binom{X}{k}$, so that for every $T \in \binom{X}{t}, |\{B \in \mathcal{B} | T \subseteq B\}| = \lambda$. The elements of $\mathcal{B}$ are called *blocks*. Elementary counting arguments show that a t-(v, k, λ) design is also an s-(v, k, μ)

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design for all $s, 0 \leq s \leq t$, where

$$\mu = \frac{\lambda\binom{v-s}{t-s}}{\binom{k-s}{t-s}}. \tag{1}$$

Since μ must be an integer, (1) yields the following well-known necessary *divisibility conditions* for the existence of a t-(v,k,λ) design:

$$\lambda\binom{v-s}{t-s} \equiv 0(\text{mod}\binom{k-s}{t-s}), \qquad 0 \leq s < t. \tag{2}$$

For any given t, k, and v, we denote by $\lambda^*(t,k,v)$ the smallest positive λ that satisfies the divisibility conditions (2).

A *large set of* t-(v,k,λ) *designs* is a partition of the complete k-uniform hypergraph $(X, \binom{X}{k})$ into t-(v,k,λ) designs, and is denoted by LS$[N](t,k,v)$, where $N = \binom{v-t}{k-t}/\lambda$ is the number of parts in the partition. A large set of t-$(v,k,\lambda^*(t,k,v))$ designs is also known as a (t,k,v)-*decomposition*.

When the underlying point set X is apparent, we identify an LS$[N](t,k,v)$ with its collection of constituent designs $\mathbb{B} = \{\mathcal{B}_i\}_{i=1}^N$. If $x \in X$, by taking the N derived designs $\mathcal{B}_i^{(x)}$ through point x, one obtains an LS$[N](t-1,k-1,v-1)$, denoted by $\mathbb{B}^{(x)}$, with underlying set $X \setminus \{x\}$,

A permutation $\gamma \in \mathcal{S}_X$ is said to be an *automorphism* of a large set $\mathbb{B} = \{\mathcal{B}_i\}_{i=1}^N$, if $\mathbb{B}^\gamma = \mathbb{B}$, that is, if $\mathcal{B}_i^\gamma \in \mathbb{B}$ for each design $\mathcal{B}_i \in \mathbb{B}$. The set of all automorphism of a large set $\mathbb{B}$ is, of course, a subgroup of $\mathcal{S}_X$ denoted by Aut$\mathbb{B}$. If G is a subgroup of Aut$\mathbb{B}$ we say that G is an *automorphism group* of $\mathbb{B}$ or that $\mathbb{B}$ is G-*invariant*. If G is an automorphism group of $\mathbb{B}$, the collection of all elements $\gamma \in G$ for which $\mathcal{B}_i^\gamma = \mathcal{B}_i$ for all $\mathcal{B}_i \in \mathbb{B}$, is a normal subgroup $G_{[\mathbb{B}]}$ of G. Note that if $\gamma \in H \leq G_{[\mathbb{B}]}$ for some automorphism group G of $\mathbb{B}$, then γ is an automorphism of each constituent design $\mathcal{B}_i \in \mathbb{B}, 1 \leq i \leq N$. In this case we say that $\mathbb{B}$ is a large set of H-*invariant* t-designs.

The purpose of this article is to construct some large sets of t-designs whose existence was previously not known. A few of these large sets are used to obtain further new large sets. One of these also gives rise to a new infinite family of large sets of 3-designs. Our approach is primarily computational and employs group actions to curb the complexity of searches. We begin in the next section with a construction for an LS[6](3, 6, 13) which many have missed.

2. KNOWN RECURSIVE CONSTRUCTIONS

Several recursive constructions for large sets of t-designs are known. The following result is implicit in the work of Khosrovshahi and Ajoodani-Namini [17], and can be derived also from earlier results of Tran [39] or Magliveras and Plambeck [26]. We supply a proof here for the sake of completeness.

Theorem 2.1. *If there exist an* LS$[M](t,k,v)$ *and an* LS$[N](t,k+1,v)$, *then there exists an* LS[gcd$(M,N)](t,k+1,v+1)$.

Proof. First observe that the existence of an LS$[M](t,k,v)$ implies the existence of an LS$[D](t,k,v)$ for any D dividing M. Hence, if there is an LS$[M](t,k,v)$ and an LS$[N](t,k+1,v)$, then we have an LS$[D](t,k+1,v)$, where $D = \text{gcd}(M,N)$.

Let $\{(X, \mathcal{A}_i)\}_{i=1}^{D}$ and $\{(X, \mathcal{B}_i)\}_{i=1}^{D}$ be an $\text{LS}[D](t, k, v)$ and an $\text{LS}[D](t, k+1, v)$, respectively. Let

$$Y = X \cup \{\infty\}, \qquad \text{and}$$

$$\mathcal{C}_i = \mathcal{B}_i \cup \{A \cup \{\infty\} | A \in \mathcal{A}_i\}, \qquad 1 \leq i \leq D,$$

where $\infty \notin X$. It is easy to show that each $(Y, \mathcal{C}_i)$ is a t-$(v+1, k+1, \binom{v-t+1}{k-t+1}/D)$ design such that $C_i \cap C_j = \emptyset$ for all $i \neq j$. Hence, $\{(Y, \mathcal{C}_i)\}_{i=1}^{D}$ is an $\text{LS}[D](t, k+1, v+1)$. □

It has been known to the first author for a long time that Theorem 2.1 can be used to construct an LS[6](3, 6, 13), which is also a (3, 6, 13)-decomposition. This fact seems not to be known. The existence of an LS[6](3, 6, 13) is quoted as being in doubt in [20] and also in the recent survey of Kreher [23].

Corollary 2.1. *There exists a* (3, 6, 13)*-decomposition.*

Proof. The existence of an LS[6](3, 5, 12) and an LS[42](3, 6, 12) has been established in [20]. Now apply Theorem 2.1 to obtain an LS[6](3, 6, 13). □

We summarize here the known results for recursive constructions of large sets of t-designs.

Theorem 2.2 (Teirlinck [38]). *For every natural number t let* $\lambda(t) = \text{lcm}\{\binom{t}{m} : m = 1, 2, \ldots, t\}, \lambda^*(t) = \text{lcm}\{1, 2, \ldots, t+1\}$, *and* $\ell(t) = \prod_{i=1}^{t} \lambda(i) \cdot \lambda^*(i)$. *Then, for all* $N > 0$, *there is an* $\text{LS}[N](t, t+1, t + N \cdot \ell(t))$.

Theorem 2.3 (Khosrovshahi and Ajoodani-Namini [17]). *If there are* $\text{LS}[N](t, t+1, v)$ *and* $\text{LS}[N](t, t+1, w)$, *then there is also an* $\text{LS}[N](t, t+1, v+w-t)$.

Theorem 2.4 (Qiu-rong Wu [41]). *If there exist large sets* $\text{LS}[N](t, k, v), \text{LS}[N](t, k, w), \text{LS}[N](k-2, k-1, v-1), \text{LS}[N](k-2, k-1, w-1)$, *then there exists a large set* $\text{LS}[N](t, k, v+w-k+1)$.

Corollary 2.2. *If there exist large sets* $\text{LS}[N](t, k, v)$, *and* $\text{LS}[N](k-2, k-1, v-1)$, *then there exist large sets* $\text{LS}[N](t, k, v + m(v-k+1))$, *for all* $m \geq 0$.

An interesting recent construction by Ajoodani-Namini [1] produces a new large set of $(t+1)$-designs, from a large set of t-designs.

Theorem 2.5 (Ajoodani-Namini [1]). *If there exists an* $\text{LS}[N](t, m, v-1)$, *and* $mN < k < (m+1)N$, *then a* $\text{LS}[N](t+1, k, Nv)$ *also exists.*

3. THE ALGORITHM

Let G be a group acting on X. Hence, G also acts on $\binom{X}{r}$ by canonical extension, for $0 \leq r \leq |X|$. Recall that a t-(v, k, λ) design $(X, \mathcal{B})$ is said to be *G-invariant* if $B^\gamma \in \mathcal{B}$ for all $B \in \mathcal{B}$ and $\gamma \in G$. Let $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_\tau$ and $\mathcal{K}_1, \mathcal{K}_2, \ldots, \mathcal{K}_\kappa$ be the orbits of $\binom{X}{t}$ and $\binom{X}{k}$ under the action of G, respectively. The *Kramer–Mesner matrix* is the $\tau \times \kappa$ matrix $A(G|X)$ whose (i, j)th entry is the quantity

$$|\{K \in \mathcal{K}_j | T \subseteq K\}|,$$

where $T \in \mathcal{T}_i$ is any arbitrarily chosen fixed representative. A well-known observation of Kramer and Mesner [22] states that there exists a G-invariant t-(v, k, λ) design $(X, \mathcal{B})$ if and only if there exists a vector $\mathbf{u} \in \{0, 1\}^\kappa$ satisfying the equation $A(G|X)\mathbf{u} = \lambda\mathbf{j}$, where $\mathbf{j}$ is the τ-dimensional vector of all ones. Our focus is on constructing not just one G-invariant t-design, but a large set of t-designs, each of which is G-invariant. We call such a combinatorial structure a *large set of G-invariant t-designs*.

Suppose U is the $\kappa \times n$ matrix whose columns are all the vectors $\mathbf{u}$ satisfying $A(G|X)\mathbf{u} = \lambda\mathbf{j}$. It is not hard to see that there exists a large set of G-invariant t-(v, k, λ) designs if and only if there exists a vector $\mathbf{w} \in \{0, 1\}^n$ such that $U\mathbf{w} = \mathbf{j}$.

Our discussion above suggests the following two-stage algorithm for constructing large sets of G-invariant t-designs:

Stage 1: Determine all solutions to $A(G|X)\mathbf{u} = \lambda\mathbf{j}$;
Stage 2: Determine whether there exists a solution to $U\mathbf{w} = \mathbf{j}$.

We implemented a simple backtracking algorithm that finds all $\{0, 1\}$-vectors $\mathbf{x}$ satisfying a general matrix equation $A\mathbf{x} = \mathbf{b}$. This implementation is employed in both stages of the above algorithm. As is typical for computational methods in combinatorial design theory, the ability to construct large sets of t-designs using our algorithm lies in the art of choosing an appropriate group G. If G is too small, then it may be infeasible to find all solutions in stage one of the algorithm. On the other hand, if G is too large, then we may not find any solution in stage two or even stage one of the algorithm.

In the next section, we demonstrate the success of our algorithm in constructing some new large sets of t-designs, by choosing G to be appropriate Frobenius groups.

4. LARGE SETS OF t-DESIGNS INVARIANT UNDER FROBENIUS GROUPS

In this section, the set X of a t-(v, k, λ) design $(X, \mathcal{B})$ is taken to be a finite field $GF(v) = \{0, 1, \ldots, v-1\}$. For each large set LS$[N](t, k, v)$, we exhibit the N t-designs comprising it. A t-design is given by a set of starter blocks, which when developed by the group G, give the blocks for the t-design.

A. An LS[11](2, 5, 13)

Let G be the Frobenius group generated by the following two permutations:

$$x \mapsto x + 1 (\text{mod } 13), \qquad \text{and} \quad x \mapsto 3x (\text{mod } 13).$$

A large set of G-invariant 2-(13, 5, 15) designs constructed using the algorithm we described in Section 3 is given in Table I.

B. An LS[11](2, 6, 13)

Let G be the Frobenius group generated by the following two permutations:

$$x \mapsto x + 1 (\text{mod } 13), \qquad \text{and} \quad x \mapsto 3x (\text{mod } 13).$$

A large set of G-invariant 2-(13, 6, 30) designs constructed using our algorithm is given in Table II.

TABLE I. LS[11](2, 5, 13) invariant under F_{13}^3

Design 1	{0, 1, 5, 6, 8}	{0, 1, 2, 5, 8}	{0, 1, 4, 5, 6}
Design 2	{0, 1, 2, 5, 6}	{0, 1, 4, 5, 7}	{0, 1, 3, 6, 8}
Design 3	{0, 1, 3, 4, 5}	{0, 1, 2, 5, 7}	{0, 1, 5, 6, 7}
Design 4	{0, 1, 4, 5, 8}	{0, 1, 3, 5, 8}	{0, 1, 5, 7, 8}
Design 5	{0, 1, 5, 8, 10}	{0, 1, 5, 8, 9}	{0, 1, 5, 7, 9}
Design 6	{0, 1, 3, 5, 9}	{0, 1, 2, 7, 8}	{0, 1, 4, 5, 9}
Design 7	{0, 1, 2, 5, 9}	{0, 1, 5, 6, 9}	{0, 1, 5, 7, 12}
Design 8	{0, 1, 3, 5, 10}	{0, 1, 2, 5, 10}	{0, 1, 3, 8, 10}
Design 9	{0, 1, 5, 7, 10}	{0, 1, 3, 6, 10}	{0, 1, 5, 6, 11}
Design 10	{0, 1, 5, 8, 11}	{0, 1, 2, 5, 11}	{0, 1, 5, 6, 12}
Design 11	{0, 1, 3, 5, 12}	{0, 1, 3, 6, 12}	{0, 1, 5, 8, 12}

Since an element g of order 3 in G fixes exactly one point, there are 6-subsets of $X =$ GF(13) fixed by g. Consequently, there are G-orbits of length 13 in $\binom{X}{6}$. More precisely, there are exactly 6 such short orbits of 6-sets. Design 11 in the above table consists of all 6 orbits of length 13 together with an additional 2 orbits of length 39. This accounts for 8 starters in the case of Design 11. All other designs in the displayed LS[11](2, 6, 13) consist of exactly 4 long orbits of 6-sets.

C. A (3, 4, 17)-Decomposition

Let G be the Frobenius group generated by the following two permutations:

$$x \mapsto x + 1 (\text{mod } 17), \qquad \text{and} \quad x \mapsto 4x (\text{mod } 17).$$

A large set of G-invariant 3-(17, 4, 2) designs constructed using our algorithm is given in Table III.

It is clear that there are 4-subsets of $X = GF(17)$ fixed by an element g of order 4 in G. These 4-sets are clearly also fixed by the involution g^2. A careful consideration of the involutions in G shows that there are 4-subsets of X fixed by an involution which are not fixed by any element of order 4. In particular, there are exactly 28 G-orbits of 4-sets of size

TABLE II. LS[11](2, 6, 13) invariant under F_{13}^3

Design 1	{0, 1, 2, 3, 5, 6}	{0, 1, 3, 5, 6, 8}	{0, 1, 2, 5, 8, 11}	{0, 1, 4, 5, 6, 11}
Design 2	{0, 1, 2, 3, 5, 7}	{0, 1, 2, 5, 6, 10}	{0, 1, 4, 5, 8, 11}	{0, 1, 5, 6, 7, 12}
Design 3	{0, 1, 2, 3, 4, 5}	{0, 1, 4, 5, 6, 8}	{0, 1, 3, 6, 8, 10}	{0, 1, 3, 5, 8, 11}
Design 4	{0, 1, 4, 5, 6, 9}	{0, 1, 2, 5, 7, 10}	{0, 1, 5, 6, 8, 11}	{0, 1, 5, 8, 9, 11}
Design 5	{0, 1, 3, 4, 5, 8}	{0, 1, 2, 5, 8, 9}	{0, 1, 5, 6, 7, 11}	{0, 1, 4, 5, 6, 12}
Design 6	{0, 1, 2, 3, 5, 8}	{0, 1, 3, 4, 5, 6}	{0, 1, 5, 6, 8, 9}	{0, 1, 3, 5, 8, 10}
Design 7	{0, 1, 5, 6, 7, 8}	{0, 1, 2, 4, 5, 6}	{0, 1, 5, 6, 8, 10}	{0, 1, 2, 5, 6, 11}
Design 8	{0, 1, 3, 4, 5, 7}	{0, 1, 2, 5, 7, 8}	{0, 1, 2, 5, 8, 10}	{0, 1, 2, 5, 6, 12}
Design 9	{0, 1, 3, 5, 6, 7}	{0, 1, 3, 4, 5, 9}	{0, 1, 4, 5, 7, 9}	{0, 1, 2, 5, 7, 9}
Design 10	{0, 1, 2, 5, 6, 8}	{0, 1, 5, 7, 8, 9}	{0, 1, 2, 5, 8, 12}	{0, 1, 5, 6, 9, 12}
Design 11	{0, 1, 4, 5, 6, 7}	{0, 1, 2, 4, 5, 8}	{0, 1, 3, 5, 7, 8}	{0, 1, 3, 4, 5, 10}
	{0, 1, 3, 6, 8, 12}	{0, 1, 3, 5, 8, 12}	{0, 1, 2, 5, 9, 12}	{0, 1, 5, 7, 10, 12}

TABLE III. LS[7](3, 4, 17) invariant under F_{17}^{4}

Design 1	{0, 2, 5, 8}	{0, 2, 3, 4}	{0, 2, 3, 7}	{0, 1, 6, 7}	{0, 2, 6, 8}	{0, 1, 7, 10}
Design 2	{0, 2, 5, 6}	{0, 2, 4, 5}	{0, 1, 2, 8}	{0, 2, 3, 9}	{0, 3, 6, 9}	{0, 1, 6, 12}
Design 3	{0, 2, 3, 5}	{0, 1, 2, 5}	{0, 1, 7, 8}	{0, 2, 6, 9}	{0, 2, 3, 11}	{0, 1, 7, 12}
Design 4	{0, 1, 2, 6}	{0, 2, 5, 7}	{0, 2, 4, 10}	{0, 2, 5, 12}	{0, 2, 3, 13}	{0, 2, 3, 16}
Design 5	{0, 1, 2, 9}	{0, 1, 4, 11}	{0, 2, 5, 13}	{0, 2, 8, 11}	{0, 1, 4, 14}	{0, 2, 5, 16}
Design 6	{0, 1, 2, 3}	{0, 1, 4, 7}	{0, 2, 3, 10}	{0, 1, 4, 10}	{0, 2, 8, 10}	{0, 2, 5, 15}
	{0, 2, 5, 14}					
Design 7	{0, 1, 4, 5}	{0, 2, 3, 6}	{0, 2, 4, 6}	{0, 1, 2, 7}	{0, 2, 5, 11}	{0, 2, 5, 9}
	{0, 1, 7, 11}					

68, 12 orbits of size 34, and 4 of size 17. Each of designs 1 to 5 is composed of 4 orbits of size 68 and 2 orbits of size 34. Each of the remaining two designs is composed of 4 orbits of size 68, one orbit of size 34 and 2 orbits of size 17. This explains the variation in the number of design starters of the LS[7](3, 4, 17).

5. SOME CONSEQUENCES

In this section we examine some of the consequences of the existence of large sets established in the previous section, and the recursive constructions presented in Section 2. In particular application of Theorem 2.1 and Corollary 2.2.

The LS[11](2, 5, 13) constructed in Section 4.A together with the LS[55](2, 4, 13) constructed by Leo Chouinard [9] gives rise to an LS[11](2, 5, 14) via Theorem 2.1. This large set is in fact a (2, 5, 14)-decomposition, whose existence was not known previously. Using the LS[11](2, 5, 14) just obtained and the LS[11](2, 4, 14) from [19], we further construct an LS[11](2, 5, 15) with $\lambda = 26$ by means again of Theorem 2.1.

The existence of a (2, 6, 13)-decomposition is still in doubt. The number of designs in a (2, 6, 13)-decomposition is 66. Therefore, short of proving the existence of a (2, 6, 13)-decomposition, the best LS[N](2, 6, 13) one can hope to construct is for $N = 6$ or $N = 11$. An LS[6](2, 6, 13) can be obtained by applying Theorem 2.1 to LS[6](2, 5, 12) and LS[42](2, 6, 12), which are known to exist (see [20]). We constructed an LS[11](2, 6, 13) in Section 4.B.

The (3, 4, 17)-decomposition constructed in Section 4.C, and its derivation, an LS[7](2, 3, 16), give rise to a new infinite family of LS[7](3, 4, 14m + 3) for all $m \geq 1$, through Theorem 2.2. Taking $m = 2$ in this family, we obtain an LS[7](3, 4, 31), which is a (3, 4, 31)-decomposition whose existence was previously not known [23].

The results of this article can be summarized as follows.

Theorem 5.1. *The following previously unknown large sets of t-designs exist*:
(1) LS[11](2, 5, v), *for* $v = 13, 14$ *and* 15;
(2) LS[11](2, 6, 13);
(3) LS[6](3, 6, 13); *and*
(4) LS[7](3, 4, 14m+ 3), *for all* $m \geq 1$.

6. CONCLUSIONS

We have constructed a few new large sets of t-designs invariant under certain Frobenius groups. These large sets spawn further new large sets by means of known recursive constructions presented in Section 2, and include a new infinite family of large sets of $3-(v,4,\lambda)$ designs. We survey the impact of these constructions by presenting an updated and expanded version of the existence table for large sets of t-designs in the Appendix. The table includes possible parameter cases for arbitrary λ in contrast to older tables which involved large sets with minimum λ only.

APPENDIX

TABLE AI. Table of Large Sets: $v \leq 18$

Parameters LS$[N](t,k,v)$	λ	Existence	Remarks
LS[2](2, 3, 6)	2	yes	Bhattacharya [3]
LS[2](2, 3, 10)	4	yes	LS[4](2, 3, 10) exists & $2\|4$
LS[2](2, 3, 14)	6	yes	Hanani [15]
LS[2](2, 3, 18)	8	yes	LS[8](2, 3, 18) exists & $2\|8$
LS[2](2, 4, 10)	14	yes	LS[14](2, 4, 10) exists & $2\|14$
LS[2](2, 4, 11)	18	yes	LS[6](2, 4, 11) exists & $2\|6$
LS[2](2, 4, 18)	60	yes	LS[10](2, 4, 18) exists & $2\|10$
LS[2](2, 5, 10)	28	yes	LS[14](2, 5, 10) exists & $2\|14$
LS[2](2, 5, 11)	42	yes	LS[42](2, 5, 11) exists & $2\|42$
LS[2](2, 5, 12)	60	yes	LS[6](2, 5, 12) exists & $2\|6$
LS[2](2, 5, 18)	280	?	
LS[2](2, 6, 12)	105	yes	LS[42](2, 6, 12) exists & $2\|42$
LS[2](2, 6, 13)	165	yes	LS[6](2, 6, 13) exists & $2\|6$
LS[2](2, 6, 18)	910	?	
LS[2](2, 7, 14)	396	yes	LS[12](2, 7, 14) exists & $2\|12$
LS[2](2, 7, 18)	2184	yes	LS[8](2, 7, 18) exists & $2\|8$
LS[2](2, 8, 18)	4004	?	
LS[2](2, 9, 18)	5720	yes	LS[10](2, 9, 18) exists & $2\|10$
LS[2](3, 4, 11)	4	yes	derivation of LS[4](4, 5, 12)
LS[2](3, 5, 11)	14	?	
LS[2](3, 5, 12)	18	yes	LS[6](3, 5, 12) exists & $2\|6$
LS[2](3, 6, 12)	42	yes	LS[42](3, 6, 12) exists & $2\|42$
LS[2](3, 6, 13)	60	yes	LS[6](3, 6, 13) exists & $2\|6$
LS[2](3, 7, 14)	165	?	
LS[2](4, 5, 12)	4	yes	Denniston [13]
LS[2](4, 6, 12)	14	?	
LS[2](4, 6, 13)	18	?	
LS[2](4, 7, 14)	60	?	
LS[2](5, 6, 13)	4	yes	derivation of LS[4](6, 7, 14)
LS[2](5, 7, 14)	18	?	
LS[2](6, 7, 14)	4	yes	Kreher and Radziszowski [24]
LS[3](2, 3, 11)	3	yes	Teirlinck [36]

TABLE AI. (Continued)

Parameters LS$[N](t,k,v)$	λ	Existence	Remarks
LS[3](2, 4, 11)	12	yes	LS[6](2, 4, 11) exists & $3\|6$
LS[3](2, 4, 12)	15	yes	LS[15](2, 4, 12) exists & $3\|15$
LS[3](2, 5, 11)	28	yes	LS[42](2, 5, 11) exists & $3\|42$
LS[3](2, 5, 12)	40	yes	LS[6](2, 5, 12) exists & $3\|6$
LS[3](2, 5, 13)	55	?	
LS[3](2, 6, 12)	70	yes	LS[42](2, 6, 12) exists & $3\|42$
LS[3](2, 6, 13)	110	yes	LS[6](2, 6, 13) exists & $3\|6$
LS[3](2, 6, 14)	165	?	
LS[3](2, 7, 14)	264	yes	LS[12](2, 7, 14) exists & $3\|12$
LS[3](2, 7, 15)	429	?	
LS[3](2, 8, 16)	1001	?	
LS[3](3, 4, 12)	3	yes	Teirlinck [36]
LS[3](3, 5, 12)	12	yes	LS[6](3, 5, 12) exists & $3\|6$
LS[3](3, 5, 13)	15	yes	Chee, Colbourn, Furino, Kreher [6]
LS[3](3, 6, 12)	28	yes	LS[42](3, 6, 12) exists & $3\|42$
LS[3](3, 6, 13)	40	yes	LS[6](3, 6, 13) exists & $3\|6$
LS[3](3, 6, 14)	55	?	
LS[3](3, 7, 14)	110	?	
LS[3](3, 7, 15)	165	?	
LS[3](3, 8, 16)	429	?	
LS[3](4, 5, 13)	3	yes	Kramer, Magliveras, and O'Brien [19]
LS[3](4, 6, 13)	12	?	
LS[3](4, 6, 14)	15	yes	Chee, Colbourn, Furino, Kreher [6]
LS[3](4, 7, 14)	40	?	
LS[3](4, 7, 15)	55	?	
LS[3](4, 8, 16)	165	?	
LS[3](5, 6, 14)	3	?	
LS[3](5, 7, 14)	12	?	
LS[3](5, 7, 15)	15	?	
LS[3](5, 8, 16)	55	?	
LS[3](6, 7, 15)	3	?	
LS[3](6, 8, 16)	15	?	
LS[3](7, 8, 16)	3	?	
LS[4](2, 3, 10)	2	yes	Teirlinck [35]
LS[4](2, 3, 18)	4	yes	LS[8](2, 3, 18) exists & $4\|8$
LS[4](2, 4, 18)	30	?	
LS[4](2, 5, 18)	140	?	
LS[4](2, 6, 18)	455	?	
LS[4](2, 7, 14)	198	yes	LS[12](2, 7, 18) exists & $4\|12$
LS[4](2, 7, 18)	1092	yes	LS[8](2, 7, 18) exists & $4\|8$
LS[5](2, 3, 7)	1	no	Cayley [5]
LS[5](2, 3, 12)	2	yes	Schreiber [30]
LS[5](2, 3, 17)	3	?	
LS[5](2, 4, 8)	3	yes	Sharry and Street [31]
LS[5](2, 4, 12)	9	yes	LS[15](2, 4, 12) exists & $5\|15$
LS[5](2, 4, 13)	11	yes	LS[55](2, 4, 13) exists & $5\|55$

TABLE AI. (Continued)

Parameters LS$[N](t, k, v)$	λ	Existence	Remarks
LS[5](2, 4, 17)	21	?	
LS[5](2, 4, 18)	24	yes	LS[10](2, 4, 18) exists & $5\|10$
LS[5](2, 8, 17)	1001	?	
LS[5](2, 9, 18)	2288	yes	LS[10](2, 9, 18) exists & $5\|10$
LS[5](3, 4, 8)	1	no	LS[5](2, 3, 7) does not exist
LS[5](3, 4, 13)	2	yes	Kramer, Magliveras, and O'Brien [19]
LS[5](3, 4, 18)	3	?	
LS[5](3, 9, 18)	1001	?	
LS[6](2, 4, 11)	6	yes	Chee, Colbourn, Furino, Kreher [6]
LS[6](2, 5, 11)	14	yes	LS[42](2, 5, 11) exists & $6\|42$
LS[6](2, 5, 12)	20	yes	LS[6](3, 5, 12) as 2-designs
LS[6](2, 6, 12)	35	yes	LS[42](2, 6, 12) exists & $6\|42$
LS[6](2, 6, 13)	55	yes	Chee, Magliveras—this article
LS[6](2, 7, 14)	132	yes	LS[12](2, 7, 14) exists & $6\|12$
LS[6](3, 5, 12)	6	yes	Kramer, Magliveras, and Stinson [20]
LS[6](3, 6, 12)	14	yes	LS[42](3, 6, 12) exists & $6\|42$
LS[6](3, 6, 13)	20	yes	Chee, Magliveras—this article
LS[6](3, 7, 14)	55	?	
LS[6](4, 6, 13)	6	?	
LS[6](4, 7, 14)	20	?	
LS[6](5, 7, 14)	6	?	
LS[7](2, 3, 9)	1	yes	Kirkman [18]
LS[7](2, 3, 16)	2	?	
LS[7](2, 4, 9)	3	yes	Kramer, Magliveras, and Stinson [20]
LS[7](2, 4, 10)	4	yes	LS[14](2, 4, 10) exists & $7\|14$
LS[7](2, 4, 16)	13	?	
LS[7](2, 4, 17)	15	?	
LS[7](2, 5, 10)	8	yes	LS[14](2, 5, 10) exists & $7\|14$
LS[7](2, 5, 11)	12	yes	LS[42](2, 5, 11) exists & $7\|42$
LS[7](2, 5, 16)	52	?	
LS[7](2, 5, 17)	65	?	
LS[7](2, 5, 18)	80	?	
LS[7](2, 6, 12)	30	yes	LS[14](2, 6, 12) exists & $7\|42$
LS[7](2, 6, 16)	143	?	
LS[7](2, 6, 17)	195	?	
LS[7](2, 6, 18)	260	?	
LS[7](3, 4, 10)	1	no	Kramer and Mesner [21]
LS[7](3, 4, 17)	2	yes	Chee, Magliveras—this article
LS[7](3, 5, 10)	3	yes	extension of LS[7](2, 4, 9)
LS[7](3, 5, 11)	4	?	
LS[7](3, 5, 17)	13	?	
LS[7](3, 5, 18)	15	?	
LS[7](3, 6, 12)	12	yes	LS[42](3, 6, 12) exists & $7\|42$
LS[7](3, 6, 17)	52	?	
LS[7](3, 6, 18)	65	?	
LS[7](4, 5, 11)	1	no	LS[7](3, 4, 10) does not exist
LS[7](4, 5, 18)	2	?	

TABLE AI. (Continued)

Parameters LS[N](t, k, v)	λ	Existence	Remarks
LS[7](4, 6, 12)	4	?	
LS[7](4, 6, 18)	13	?	
LS[7](5, 6, 12)	1	no	LS[7](3, 4, 10) does not exist
LS[8](2, 3, 18)	2	?	
LS[8](2, 7, 18)	546	?	
LS[10](2, 4, 18)	12	?	
LS[10](2, 9, 18)	1144	?	
LS[11](2, 3, 13)	1	yes	Denniston [12]
LS[11](2, 4, 13)	5	yes	LS[55](2, 4, 13) exists & $11\|55$
LS[11](2, 4, 14)	6	yes	Kramer, Magliveras, and O'Brien [19]
LS[11](2, 5, 13)	15	yes	Chee, Magliveras—this article
LS[11](2, 5, 14)	20	yes	Chee, Magliveras—this article
LS[11](2, 5, 15)	26	yes	Chee, Magliveras—this article
LS[11](2, 6, 13)	30	yes	Chee, Magliveras—this article
LS[11](2, 6, 14)	45	?	
LS[11](2, 6, 15)	65	?	
LS[11](2, 6, 16)	91	?	
LS[11](2, 7, 14)	72	?	
LS[11](2, 7, 15)	117	?	
LS[11](2, 7, 16)	182	?	
LS[11](2, 7, 17)	273	?	
LS[11](2, 8, 16)	273	?	
LS[11](2, 8, 17)	455	?	
LS[11](2, 8, 18)	728	?	
LS[11](2, 9, 18)	1040	?	
LS[11](3, 4, 14)	1	?	
LS[11](3, 5, 14)	5	?	
LS[11](3, 5, 15)	6	?	
LS[11](3, 6, 14)	15	?	
LS[11](3, 6, 15)	20	?	
LS[11](3, 6, 16)	26	?	
LS[11](3, 7, 14)	30	?	
LS[11](3, 7, 15)	45	?	
LS[11](3, 7, 16)	65	?	
LS[11](3, 7, 17)	91	?	
LS[11](3, 8, 16)	117	?	
LS[11](3, 8, 17)	182	?	
LS[11](3, 8, 18)	273	?	
LS[11](3, 9, 18)	455	?	
LS[11](4, 5, 15)	1	no*	Mendelsohn and Hung [27]
LS[11](4, 6, 15)	5	?	
LS[11](4, 6, 16)	6	?	
LS[11](4, 7, 15)	15	?	
LS[11](4, 7, 16)	20	?	

TABLE AI. (Continued)

Parameters LS$[N](t, k, v)$	λ	Existence	Remarks
LS[11](4, 7, 17)	26	?	
LS[11](4, 8, 16)	45	?	
LS[11](4, 8, 17)	65	?	
LS[11](4, 8, 18)	91	?	
LS[11](4, 9, 18)	182	?	
LS[11](5, 6, 16)	1	no	LS[11](4, 5, 15) does not exist
LS[11](5, 7, 16)	5	?	
LS[11](5, 7, 17)	6	?	
LS[11](5, 8, 16)	15	?	
LS[11](5, 8, 17)	20	?	
LS[11](5, 8, 18)	26	?	
LS[11](5, 9, 18)	65	?	
LS[11](6, 7, 17)	1	no	LS[11](5, 6, 16) does not exist
LS[11](6, 8, 17)	5	?	
LS[11](6, 8, 18)	6	?	
LS[11](6, 9, 18)	20	?	
LS[11](7, 8, 18)	1	no	LS[11](6, 7, 17) does not exist
LS[11](7, 9, 18)	5	?	
LS[12](2, 7, 14)	66	?	
LS[13](2, 3, 15)	1	yes	Denniston [12]
LS[13](2, 4, 15)	6	?	
LS[13](2, 4, 16)	7	?	
LS[13](2, 5, 15)	22	?	
LS[13](2, 5, 16)	28	?	
LS[13](2, 5, 17)	35	?	
LS[13](2, 6, 15)	55	?	
LS[13](2, 6, 16)	77	?	
LS[13](2, 6, 17)	105	?	
LS[13](2, 6, 18)	140	?	
LS[13](2, 7, 15)	99	?	
LS[13](2, 7, 16)	154	?	
LS[13](2, 7, 17)	231	?	
LS[13](2, 7, 18)	336	?	
LS[13](2, 8, 16)	231	?	
LS[13](2, 8, 17)	385	?	
LS[13](2, 8, 18)	616	?	
LS[13](2, 9, 18)	880	?	
LS[13](3, 4, 16)	1	?	
LS[13](3, 5, 16)	6	?	
LS[13](3, 5, 17)	7	?	
LS[13](3, 6, 16)	22	?	
LS[13](3, 6, 17)	28	?	
LS[13](3, 6, 18)	35	?	
LS[13](3, 7, 16)	55	?	
LS[13](3, 7, 17)	77	?	
LS[13](3, 7, 18)	105	?	

TABLE AI. (Continued)

Parameters LS$[N](t,k,v)$	λ	Existence	Remarks
LS[13](3, 8, 16)	99	?	
LS[13](3, 8, 17)	154	?	
LS[13](3, 8, 18)	231	?	
LS[13](3, 9, 18)	385	?	
LS[13](4, 5, 17)	1	?	
LS[13](4, 6, 17)	6	?	
LS[13](4, 6, 18)	7	?	
LS[13](4, 7, 17)	22	?	
LS[13](4, 7, 17)	28	?	
LS[13](4, 8, 17)	55	?	
LS[13](4, 8, 18)	77	?	
LS[13](4, 9, 18)	154	?	
LS[13](5, 6, 18)	1	?	
LS[13](5, 7, 18)	6	?	
LS[13](5, 8, 18)	22	?	
LS[13](5, 9, 18)	55	?	
LS[14](2, 4, 10)	2	yes	Kramer, Magliveras, and Stinson [20]
LS[14](2, 5, 10)	4	yes	Kramer, Magliveras, and Stinson [20]
LS[14](2, 5, 11)	6	yes	LS[42](2, 5, 11) exists & $14 \mid 42$
LS[14](2, 5, 18)	40	?	
LS[14](2, 6, 12)	15	yes	LS[42](2, 6, 12) exists & $14 \mid 42$
LS[14](2, 6, 18)	130	?	
LS[14](3, 5, 11)	2	no*	Oberschelp [29] and Dehon [11]
LS[14](3, 6, 12)	6	?	
LS[14](4, 6, 12)	2	no*	LS[14](3, 5, 11) does not exist
LS[15](2, 4, 12)	3	yes	Kramer, Magliveras, and Stinson [20]
LS[20](2, 4, 18)	6	?	
LS[21](2, 5, 11)	4	yes	LS[42](2, 5, 11) exists & $21 \mid 42$
LS[21](2, 6, 12)	10	yes	LS[42](2, 6, 12) exists & $21 \mid 42$
LS[21](3, 6, 12)	4	yes	LS[42](3, 6, 12) exists & $21 \mid 42$
LS[22](2, 6, 13)	15	?	
LS[22](2, 7, 14)	36	?	
LS[22](2, 8, 18)	364	?	
LS[22](2, 9, 18)	520	?	
LS[22](3, 7, 14)	15	?	
LS[26](2, 6, 18)	70	?	
LS[26](2, 7, 18)	168	?	
LS[26](2, 8, 18)	308	?	
LS[26](2, 9, 18)	440	?	
LS[28](2, 5, 18)	20	?	
LS[28](2, 6, 18)	65	?	
LS[33](2, 5, 13)	5	?	See [19]
LS[33](2, 6, 13)	10	?	
LS[33](2, 6, 14)	15	?	

TABLE AI. (Continued)

Parameters LS$[N](t, k, v)$	λ	Existence	Remarks
LS[33](2, 7, 14)	24	?	
LS[33](2, 7, 15)	39	?	
LS[33](2, 8, 16)	91	?	
LS[33](3, 6, 14)	5	?	
LS[33](3, 7, 14)	10	?	
LS[33](3, 7, 15)	15	?	
LS[33](3, 8, 16)	39	?	
LS[33](4, 7, 15)	5	?	
LS[33](4, 8, 16)	15	?	
LS[33](5, 8, 16)	5	?	
LS[35](2, 4, 17)	3	?	
LS[39](2, 7, 15)	33	?	
LS[39](2, 8, 16)	77	?	
LS[39](3, 8, 16)	33	?	
LS[42](2, 5, 11)	2	yes	Kramer, Magliveras, and Stinson [20]
LS[42](2, 6, 12)	5	yes	LS[42](3, 6, 12) as 2-designs
LS[42](3, 6, 12)	2	yes	extension of LS[42](2, 5, 11)
LS[44](2, 7, 14)	18	?	
LS[52](2, 6, 18)	35	?	
LS[52](2, 7, 18)	84	?	
LS[55](2, 4, 13)	1	yes	Chouinard [9]
LS[55](2, 8, 17)	91	?	
LS[55](2, 9, 18)	208	?	
LS[55](3, 9, 18)	91	?	
LS[65](2, 8, 17)	77	?	
LS[65](2, 9, 18)	176	?	
LS[65](3, 9, 18)	77	?	
LS[66](2, 6, 13)	5	?	
LS[66](2, 7, 14)	12	?	
LS[66](3, 7, 14)	5	?	
LS[77](2, 6, 16)	13	?	
LS[91](2, 4, 16)	1	?	
LS[91](2, 5, 16)	4	?	
LS[91](2, 5, 17)	5	?	
LS[91](2, 6, 16)	11	?	
LS[91](2, 6, 17)	15	?	
LS[91](2, 6, 18)	20	?	
LS[91](3, 5, 17)	1	?	
LS[91](3, 6, 17)	4	?	
LS[91](3, 6, 18)	5	?	
LS[91](4, 6, 18)	1	?	
LS[104](2, 7, 18)	42	?	

TABLE AI. (Continued)

Parameters $\mathrm{LS}[N](t,k,v)$	λ	Existence	Remarks
LS[110](2, 9, 18)	104	?	
LS[130](2, 9, 18)	88	?	
LS[132](2, 7, 14)	6	?	
LS[143](2, 5, 15)	2	?	
LS[143](2, 6, 15)	5	?	
LS[143](2, 6, 16)	7	?	
LS[143](2, 7, 15)	9	?	
LS[143](2, 7, 16)	14	?	
LS[143](2, 7, 17)	21	?	
LS[143](2, 8, 16)	21	?	
LS[143](2, 8, 17)	35	?	
LS[143](2, 8, 18)	56	?	
LS[143](2, 9, 18)	80	?	
LS[143](3, 6, 16)	2	?	
LS[143](3, 7, 16)	5	?	
LS[143](3, 7, 17)	7	?	
LS[143](3, 8, 16)	9	?	
LS[143](3, 8, 17)	14	?	
LS[143](3, 8, 18)	21	?	
LS[143](3, 9, 18)	35	?	
LS[143](4, 7, 17)	2	?	
LS[143](4, 8, 17)	5	?	
LS[143](4, 8, 18)	7	?	
LS[143](4, 9, 18)	14	?	
LS[143](5, 8, 18)	2	?	
LS[143](5, 9, 18)	5	?	
LS[182](2, 6, 18)	10	?	
LS[286](2, 8, 18)	28	?	
LS[286](2, 9, 18)	40	?	
LS[364](2, 6, 18)	5	?	
LS[429](2, 7, 15)	3	?	
LS[429](2, 8, 16)	7	?	
LS[429](3, 8, 16)	3	?	
LS[715](2, 8, 17)	7	?	
LS[715](2, 9, 18)	16	?	
LS[715](3, 9, 18)	7	?	
LS[1001](2, 6, 16)	1	?	
LS[1430](2, 9, 18)	8	?	

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