ON A PROBLEM OF HARTMAN AND HEINRICH CONCERNING PAIRWISE BALANCED DESIGNS WITH HOLES*

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Abstract. We consider the problem of constructing pairwise balanced designs of order v with a hole of size k. This problem was addressed by Hartman and Heinrich who gave an almost complete solution. To date, there remain fifteen unresolved cases. In this paper, we construct designs settling all of these.

Key words. pairwise balanced designs, hillclimbing

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1. Introduction. Let $\mathcal{K}$ be a set of positive integers. A *pairwise balanced design* (PBD) of *order* v with *block sizes* from $\mathcal{K}$, denoted PBD$(v, \mathcal{K})$, is a pair $(\mathcal{X}, \mathcal{B})$, where $\mathcal{X}$ is a finite set of v *points* and $\mathcal{B}$ is a set of subsets of $\mathcal{X}$, called *blocks*, with the property that $|B| \in \mathcal{K}$ for all $B \in \mathcal{B}$, and every 2-subset of $\mathcal{X}$ appears in precisely one block. PBD$(v, \mathcal{K} \cup \{k^*\})$ is a notation for a PBD of order v with one block of size k and all other blocks having sizes in $\mathcal{K}$. A PBD$(v, \mathcal{K} \cup \{k^*\})$ is also known as a PBD$(v, \mathcal{K})$ with a *hole* of size k.

Let $\mathbf{Z}_{\geq 3}$ be the set of all integers that are at least three. The problem of constructing designs PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$ was considered by Hartman and Heinrich in [2], where the following result is established.

THEOREM 1.1. *A PBD*$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$ *exists if and only if* $v \geq 2k+1$ *except when*

(i) $v = 2k+1$ *and* $k \equiv 0 \pmod 2$;
(ii) $v = 2k+2$ *and* $k \not\equiv 4 \pmod 6$, $k > 1$;
(iii) $v = 2k+3$ *and* $k \equiv 0 \pmod 2$, $k > 6$;
(iv) $(v, k) \in \{(7, 2), (8, 2), (9, 2), (10, 2), (11, 4), (12, 2), (13, 2)\}$, *and possibly when* $(v, k) \in \mathcal{P} = \{(17, 6), (21, 8), (26, 9), (28, 11), (29, 10), (29, 12), (30, 11), (33, 14), (35, 12), (37, 14), (38, 13), (39, 14), (42, 17), (47, 18), (49, 20), (55, 20)\}$.

The possible exception $(v, k) = (17, 6)$ in Theorem 1.1 was subsequently removed by Heathcote [3] who showed that there cannot exist a PBD$(17, \mathbf{Z}_{\geq 3} \cup \{6^*\})$. Since then, there remain fifteen pairs $(v, k) \in \mathcal{P}$ for which the existence of a PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$ is undetermined. In this note, we construct PBDs settling the problem for all of the pairs in $\mathcal{P}$.

The strategy we used in constructing a PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$ $(\mathcal{X}, \mathcal{B})$ is to completely specify the set of blocks $\mathcal{A} \subseteq \mathcal{B}$ with sizes greater than three, that is, $\mathcal{A} = \{B \in \mathcal{B} \mid |B| \geq 4\}$. Following [1], we call the partial design $(\mathcal{X}, \mathcal{A})$ the *prestructure* of the PBD. The remaining blocks of size three (*triples*) are then filled in by a variant of Stinson's hillclimbing algorithm [4] similar to the one described in [1].

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(v,k)	$(21,8)$	$(26,9)$	$(28,11)$	$(29,10)$	$(29,12)$	$(30,11)$	$(33,14)$	$(35,12)$
Blocks in prestructure	aijkl	amouz	amxyz	alszC	erstv	anuvw	auvwx	aopqr
	bimno	ajkl	ilvAB	akpu	amqx	klmzA	bAEFG	amsy
	ampq	bjmn	alot	bksv	bmry	alot	aotA	bmtA
	anrs	cjop	blps	clpw	cmsz	blpu	bouC	cnuC
	aotu	dkqs	clqu	dlqu	dmtA	clqv	covE	dnvz
	bjpr	emqt	dmpv	emrv	enqy	dmrw	dowG	eowB
	bkqt	fquv	emqs	fmsw	fnrx	empx	epxB	foxD
	blsu	gkrw	fmrw	gnqx	gosB	fmqy	fpyD	gpsA
	cnpu	hmrx	gnpx	hnty	hotC	gnrz	gpzF	hptC
	doqr	iryz	hnqy	iory	ipvz	hnpA	hptE	iquz
	emst		inrz	jotx	jpwA	inqB	iquG	jqvB
	fkps		jorA		kuvB	jorC	jqvB	krwD
	gjqu		kosB		luwC	kosD	krwD	lrxy
	hlrt						lrxF	
							msyA	
							nszC	
(v,k)	$(37,14)$	$(38,13)$	$(39,14)$	$(42,17)$	$(47,18)$	$(49,20)$	$(55,20)$	
Blocks in prestructure	auvwx	zABCD	auvwx	rstuv	KLMNO	DEFGH	DEFGH	
	bAEFG	anrz	bAEFG	LMNOP	asBK	STUVW	STUVW	
	aotA	bnsA	aotA	arwF	bsCM	auDN	auDN	
	bouC	cntB	bouC	brxC	ctDO	buEP	buEP	
	covE	dorC	covE	cryB	dtEQ	cuFR	cuFR	
	dowG	eosD	dowG	dszC	euFS	duGT	duGT	
	epxB	fotE	epxB	esxB	fuGL	evHV	evHV	
	fpyD	gpuF	fpyD	fsyG	gvHN	fvIO	fvIO	
	gpzF	hpsG	gpzF	gtBH	hvIP	gwJQ	gwJQ	
	hptE	iptH	hptE	htxI	iwJR	hwKS	hwKS	
	iquG	jqvI	iquG	ityC	jwBM	ixLU	ixLU	
	jqvB	kqwJ	jqvB	juzA	kxCO	jxMW	jxMW	
	krwD	lqxK	krwD	kuDJ	lxDQ	kyDP	kyDP	
	lrxF	mryL	lrxF	luEK	myES	lyER	lyER	
	msyA		msyA	mvzL	nyFL	mzFT	mzFT	
	nszC		nszC	nvAM	ozGN	nzGV	nzGV	
				ovDN	pzHP	oAHO	oAHO	
				pwDO	qAIR	pAIQ	pAIQ	
				qwAP	rAJK	qBJS	qBJS	
						rBKU	rBKU	
						sCLW	sCLW	
						tCMN	tCMN	

TABLE 2.1. Prestructures for PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$

2. Prestructures. The most difficult task in the construction of PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$ is the determination of suitable prestructures. The prestructures $(\mathcal{X}, \mathcal{A})$ used in this paper are constructed manually, taking into account the following elementary conditions that must be satisfied:

(i) $\sum_{A \in \mathcal{A}} \binom{|A|}{2} \equiv \binom{v}{2} \pmod 3$;

(ii) for every $x \in \mathcal{X}$, $\sum_{A \in \mathcal{A} | x \in A} (|A| - 1) \equiv v - 1 \pmod 2$.

In Table 2.1, we give prestructures of designs PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$ for which the hillclimbing algorithm succeeds in completing them to PBDs. In each case, the prestructure consists of only one block of size k, and the remaining blocks have sizes four and five. The point-set of a PBD of order v is taken to be the set consisting of the first v elements of $P = \{\mathtt{a}, \mathtt{b}, \ldots, \mathtt{z}, \mathtt{A}, \mathtt{B}, \ldots, \mathtt{Z}, \mathtt{1}, \mathtt{2}, \mathtt{3}\}$. The block of size k in each prestructure is the set consisting of the first k elements of P, and we omit it from the listing in Table 2.1.

Given these prestructures, it is easy to complete them with triples to PBDs using hillclimbing. Our program, running on a DEC 2000 4/200 Alpha system, took less than two seconds on the largest design. For the sake of completeness, we include in the Appendix the triples required to complete each prestructure to the desired PBD.

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Appendix. Listing of Triples. We exhibit here the set of triples required to complete each of the prestructures in Table 2.1 to the desired PBD.

$(v, k) = (21, 8)$:

cit cjo ckr clm cqs dis djm dku dlp dnt eip ejn eko elq eru fiu fjt flo fmr fnq
gir gkm gln gos gpt hiq hjs hkn hmu hop

$(v, k) = (26, 9)$:

dlr itu apt ckn gtx boy kmv anr fkp duy blt erv aqw fnw iko gms dwx fjr bqx bkz
buw hpw asy lnx imw pxz gno dov eny hjt hlq avx hnz gpy cvw jxy elo ewz otw dnt
lwy pru glu gqz eju clm fls npq gjv fox stv cux hvy oqr eps ilp djz hos ijq jsw
isx kty ekx bpv hku brs ftz dmp lvz cqy fmy crt inv csz nsu

$(v, k) = (28, 11)$:

bno jlm fst dwz isw hpt fpq krt elz bmA kmu jsz dlx hmo env rsx dns juv fox byB
goz kln anB hlr btz twA ery kyA fly eow nuw aqw cop gsA auA kqz etu fnA epA fvz
apr bwx hxA jqx exB asv dty jwy crv ipy kvx bqv bru dqA qtB imt hzB guy dou jnt
iux gtv kpw gmB jpB ioq cwB csy glw cmn ctx fuB hsu hvw ovy puz czA gqr drB

$(v, k) = (29, 10)$:

brw iBC gvA dpv tvw csx kln axA flr krA hkz fyA hmC bxy mux gpt hqw buA hlv iuz
rst gkm ctu jly cko jvz cnr dwy imn jqs moB ekq grC kyB guw jmp anv esy boq cmq
iqv jkC ikw qzB bmz isA osu aqr fov cvB dns nwC wxz fxB gsB ipx fkt jru uvy jnB
fqC atB lmA jwA hoA amy eop ewB czA hrx dmt drz euC pqy aow blB vxC dkx doC elx
cyC enA noz btC etz fpz gyz bnp fnu glo prB dAB hps huB pAC ilt qtA

$(v, k) = (29, 12)$:

asA gmw kmn gnt guy hvy anC dyC hqA ewz ftw fvC btz coy lxB inA dsw jrC bps fou
grA itu imo awy eAB pyB cqC fzA iqr dru eux hnw iwB gzC lns hpr hsu dqv jnv lqz
cnp isC gpq dpx kpC fmp cuA aov bno doz krw apu lpt jox ctx kyz nuz jzB jty koA
arz bvA jqs cvw ksx gvx ixy fqB jmu emC oqw crB atB lyA xAC fsy bwx lor eop bqu
kqt hxz bBC dnB lmv hmB

$(v, k) = (30, 11)$:

bwD fow gou pvC bsA fvB gqA dpD ewz hrv jsw fuD ktu jvA etB crt jpz uxA dyz ayA
cwA dlx noy fpt ioA kqx qtC fAC bnt frx aqz euy ipy bvx jty enD axC kwB hqu elC
tAD erA eqs dAB mst oxB ryD amD wxy gtw iwC eov iru dtv duC jqD gsv csu doq hmo
isx hBD bqr dns arB vzD imv knC pqw boz gxD fln hlw kvy fsz gly cmn cCD kpr hsy
ilD bmB jmu lrs aps cop hzC sBC htx byC itz jlB gpB jnx gmC cyB cxz uzB

$(v, k) = (33, 14)$:

fuE lop hsB kst goq aqF ctw cpq dAD drs esF byB azD ioz moF jpu mqC eDG cxD npG
jxC cCF bqw nox cyG koB isv jzA huF aps mxG fCG gry crA fos kvF nuD iBE hxz dpC
ixy ayC csu brz fwF kqx fAB kuy gBD dqt ipr dvy jtG mpw mzE kCE eAC hrG lDE muB
mvD frv ewz kzG gsE itF dBF nvA duz gtu hvC czB lvz nyF dxE iwA jDF eqE tyz aBG
gwC hoD gvG kpA jyE bpv arE hqA etv ftx lqy mrt ltC bsx lwB btD nwE hwy ntB jsw
nqr fqz qsD eoy luA eru jor gxA lsG iCD rBC

$(v, k) = (35, 12)$:

puy gno nty gyE eqE wxG jnG qxF tzB jAC hFI ixA gzH cqD rEG lns boG jru qtH hnw
jDH vAI fzC knE lqI bBI dyD crA oCF hrH fwI avx epF rsF ezD gvD fGH tEF ctw jyI
ist cpv emC aCD brv etv awz kux muv cHI euH doE auE sEI mwH fnr hyB gmx dmI jxE
nAF bsu fqA duB dpG hAE bwy fmp fvE dtx xBH hov drC erI huG irB wCE lvC guF kpH
fsB csx bxC ozI kvF kmB yzG qCG kqy luw dFH mrz esG ltD kCI ioH bpz ouA fyF anH
hsD BDE inI dqs bEH jpw gGI eyA uDI hmq enx iyC cBF dwA lzF lpE cmG aAB bnq lBG
lmo ipD ftu imE coy kos ivG npB gqw pxI atI jot jsz ADG svw ktG vyH sCH bDF aFG

gBC hxz grt mnD czE jmF iwF kzA lAH

$(v,k) = (37,14)$:

jEH fvK qtK lCK eqF iAC BCF cFI jxy yIK bsx yEJ ntD nwE iwy eor kpC evH ayG cCH
gqA oBJ grG lwI aCE los eCI gxC kvy lpv cpq nHJ bBH koz euy dAI ewJ gsB qxD wAK
kGH hqw btw qsI nuI gwH asD iEI isJ itv mxG foq nox ezA aqH txH bpr est hsF lyH
fuz bqy svG dtu jDJ crt dBE cyz eDG jsK fsE fAH jwC gvD hxK cxA gty kFK csw jrI
ksu nGK cGJ kxE guE hCD frJ cuK aFJ nvF lAD dps kqJ wzB fCG mtF iFH dvC aBI jpG
npA rvA eEK dqz moD ftB gJK nyB oHK hzH irB dyF mHI ryC fwF ktI hAJ iop mpw hvI
bvJ tCJ arz mvz hru jtz iDK zDE hBG nqr hoy lqE luB rsH mrE pIJ mBK drK bzK ixz
joF goI apK lzJ puH bDI dxJ mqC dDH muJ fxI kAB cBD juA uDF ltG zGI

$(v,k) = (38,13)$:

jpr iBE hwC rsH uvH dyE avD bxI jCF tvx yIJ fxD AGL joL jnE iyK mCH dsJ goA kuA
gDK tCI gEI vGJ aCG fny bqC uwG oyF gnG knC gzJ lvC apK oBJ hIK irw mDE lyz fvw
pvL htu cuz wDI equ dxL grv iCJ cpC jxJ jtw dpB stz nwK dqz bvy CEL aox suK hJL
mpx iov aHI iux noI lBL evz inL muB gqH bBH lwF hnv qrB gtL qyA dvF EHK koH lou
mvA hrD kzL DHL hxA boK fFI asL nxH lpA uEJ hFH gxB isI mFJ zFK ltJ krK iAF dnu
qsF enF xyC lnD mnq iqD aEF lsE pyD msw lGH uIL btG mzI moG crA jAK cxE gwy kvE
fqG fsB bzE cqL etA cDF npJ jsy erx bru gsC hoz epE fKL auy rtF dtD juD ksx kty
kpI kDG ewL wAE opq eBI fuC aAJ kBF eyG hqE rEG xFG dAI bDJ fAH izG eCK vBK dGK
hyB frJ dwH jBG wxz fpz csv cow cyH awB mtK aqt jzH eHJ cGI cJK lrI bpw bFL

$(v,k) = (39,14)$:

fzI cBG eqt ksB hBM iDE lyE gLM aqC gEH HIL bpv wyL drz kuF dvM jyI nBE cJM koJ
sGM tuz aBD jtH hCL aEL kyG gyB cyz erv uEM ctw fAJ nuK dFH nIJ fsx vHJ jwJ pwI
eyC cFL ktx cAK kEK dsE eos cCH mxE kHM fBL gDI cxI huy arG fvF mqH dxJ ioI GHK
frE xAC eGL uAB euJ gCK svI bBI moL fCG nwA iFK joF mBF dqy zBH gsw cqD dAD mrJ
bst xDL azJ hqx jpA zKL nry iAM dtI kzA hJK gox lwz iwH apH loM opq zDG kqI bwM
luI juD psJ bDH vyK wBK ixz hrH lsH ayF byJ nxH gGJ csu aIM qsF ipL rtK gqA jsL
cpr dpK jrC eAH CEI jxG jzE hwF mvz noD nvG brL ewE ntF gru ltB bxK hsD irs mDM
orB fuH npM dBC iBJ hoz lpG mtG CFM lAL bqz ftM tCD lvD qEJ kpC eDK mpu qrM hvA
hGI lCJ nqL tJL foK fqw mwC rAI ity duL gtv jKM ivC kvL xyM eFI mIK DFJ ezM oyH
asK lqK

$(v,k) = (42,17)$:

bFN wKN jIN euN dFL ewL eDF kzP nFP qyM pyK eEM iwH fFI dwI aDG cxF gsD owJ gzJ
irD evy hHO ouL pEJ orP dyP hGK CGJ atO eJP BJM ivJ hJL cJO drE grK qKO muO nsE
AEH fEN yHL fwB jxJ duG gAC nCN kvE psI ksw guw GIO quF gyN ctz hBP ixN cDK nuH
hrz lzF oxz dtM otE erH nGL iAK nrO kBK ezK zEI cCL bAG qzD fKL qrN ktN ADI oyA
dBD jyF gEO fDP nIJ gvP xDH pzH nyD cHN aKM hAF nzB lrA lwC huy ftA kxM izG pAB
mCD ptP oFO lGM avC hDE aHP qBC hCM qHI osM dvK mwE iIM mtF eCI BFG oGH qxE jsO
jvB aAJ lIP byE ntK iBO mxA lvx etG pxL cuM nwx eAO aBE jDM bDL jtw bzO kyI lBL
fzM hvw krG qvG mGP fux xKP ltD cvI jHK jCP bwM gIL bsP gFM jrL pGN kFH kAL dHJ
csA buB cEP iuP prM isF FJK qsJ oBI azN dxO msK bIK qtL puC mrI mBN wyz oCK gxG
jEG asL myJ cwG mHM fCH CEF bvH frJ btJ kCO pvF iEL lsH axy auI hsN lJN dAN fvO
lyO

$(v,k) = (47,18)$:

iFU fEK quv uyQ ptu dvG juH fxF jtU hOR dMS CFK bHR nAQ gwK lLR nIT bxI fyI cuE
gRS lsS dCD jCQ lFG ayO mvA dwA mwP AFO rFQ ksu kLQ gBG ouO btG dsJ aDM kwN kEP
aHU nsw CGI hMT ktH gJM dyR jEJ cGM iGS mFM hFH fBR fsz gAT mIN bAD bvE hwQ mtJ
nux lCJ pOS byK nSU rCS qKP rzR HJS tBC qCN lHI GKU vwL oKS cJL vyJ kGR pyN qMQ
lOP fwH vzO rsL pCE rDG aEG oIJ gtI rxP muD nCH hzC pwF bNS uzJ kyA pxJ qLT ENR

pAB nzM qwO isP lAM eBH eIO xBS ltK dzB rEI hBE hJU wyU mxH jvD uCR aJN nDP eyD
psQ iHO evR aAS gxL buB wIS msT huA bwT avF iuT azI iKQ oyH otF owD jIL bzL pMR
eMU cAU pvU BIQ fAN jFR ftM gDF kST kDK eCL lwE ivB mGO oAC iAL guU kBJ jNT dFI
GPQ kIU ovQ cHK yMP cvS fQS aCT EFT qFJ tAP nEO lvT mzQ pDL mBL cxy qxG nvK qsH
ixz HQT awx rNU rHM oxM rBO aQR AGH zDS sDR svx ruw gzE ntR lzU eJQ iyC hDN eKT
oBP DEH rtv txT jzK cBF oRT mKR kzF atL jyG twz dHL est eNP dOU oEL bFP iEM cNQ
csI cPR gsy dxN fOT mCU dPT hxK jPS LPU qEU ewG bJO hty czT osU cwC ryT gOQ kvM
nBN gCP exE jsO xRU qtS bQU f! DU fvC hsG qBD itN DJT pIK hLS ezA iDI auP nGJ uIM
BTU duK jxA luN qyz pGT fJP lyB sFN sAE

$(v,k) = (49,20)$:

fGW puL cKL dxH fxE rwM hOU izE cHQ gAG lHJ fwF dJW iMR txO sDS tEV sxG pMU lzN
oFI pJT ozW dvN aFJ rLN ALM sKP iHS mxD qzR bIR fzD jQU kBH eAJ pCG lFO nFP guy
bAN rHP quV NOP ruJ jwy eFK qFN tDQ lGP iPV iKN kuW bHU pvy ezU bzM fAT kvK bxC
oEQ kGO tBF aKO szA yFL dAK cwD fRV azQ byS nDI iDJ fyB iCT dzL sHR hGN iOW ovR
ivB gDM lAS eDW rzC oNU pxF EOS qIU gxB euO fPQ cOT kIM eCP fuS jJO dDO iwG tyA
mOQ cxy bFW zIP twH fCU nRT ewT nMO cAV nwU hFM lvx bvJ suM dMV czB sJU hxI oDV
nCQ xNQ tRS kNS mwE lCV myK jKR lDU iAF jzH hyz oGK tJK mGJ bDK nuK sBE QRW syQ
cPU eLR pDR dyU jEN bBO mRU eGQ gNT vwz eBI eyN qLO qKQ mIS IJV jCF swO nEL lMT
fJN hAE pzS JLP jvA rvF tGU gHW tuz qDT cEJ xzK pOV oxP nAW yCO uxA rAD nxJ nvS
qHM oyT cCI pBP rOR kzJ hvD lLQ avC juI svT fKM nBN tIW oBM yJM huC BGR gKV bwV
kET muH mMP aAR dFS aEU cMS gIL jGS qCE IKT aGM mAC rQS dBQ aPS ouw luB uvU pHN
jDL dEI cvG nyH hPW axV kAU gvE bGL sIN oLS kwC wNR hJR dCR wAB CHK dwP lKW jPT
rxT hHT jBV ayW qyG qAP awL m! vL tLT lwI mBW bQT fHL kxR kLV hQV qwx aBT pEK oCJ
gFU pwW iuQ sFV ryV kFQ qvW gzO cNW mNV vMQ iyI hBL eEM rEW tvP gPR exS BCD gCS
aHI rGI

$(v,k) = (55,20)$:

LR2 lvG ayA ALM oLZ fx1 rDZ pTZ axB jHQ hXZ pRY bCH uvz gO3 fwG dEO iyI sKZ guY
oRW aVZ av3 hT3 kuB rxO eC3 aPT yzJ eZ1 lN1 oDQ uxA lDW kKL hEM gAP jET lMX qAG
pHS sP1 wAY S12 fAE lAS oCP bDT bK2 hy1 nuL iMS nR1 INX dJL cTX nQX dDV sDI iRX
bxS txZ sHR DM3 cJP nBT NPV tzW dKN qv1 LP3 tV1 rNR lHP puw eOU rHL nPS JO1 KPQ
zRS cQ2 fCR tuK iOV jvS wNO ezQ qPU dy3 iwF sBM mxy kJU oxF pvC tIT bI3 bJR ACU
JV2 gFK fuX jOY hxP qN3 jAB cLV IPR dIS jDJ lJT CEI vPX nFW ow3 dB1 k13 MQ1 uSZ
tX2 lwz CJK cMZ hvQ sOS fB2 fKV xGI QY3 cBC oEK bGQ mOQ hAN mIJ tDS jI1 rzC pz1
eBN eDK pEV yQU fzD jCF qFO nMO swE nx3 fHT aKW quQ eFJ vxD luV qyW DX1 rvY kxE
wxT yCY hGU qDL iG3 rA2 BGR KR3 ew2 cxK sAV tvF aXY FM2 hu2 BWY ART jyL nAZ mvL
EL1 gDR cGO bLX hJW yHK kwW xQV hCV tEQ aGJ twL dFP lxC cIU BX3 ou1 bvM cW1 tyO
nwC zKY cwD kOX oyB bzN kzH dAX cyN exR hzB sTY dzM yFZ fNS zE3 nvy bA1 uCO rWX
LOT aFI oGS tRU nUY dwU jUX pOP aEU hHI BDO gU1 iCD kMT sxz bwB nEN jwV py2 hDY
aLS eyT tA3 bYZ hOR eEY iPY g! EW aw1 aH2 oJN vW2 vwZ azO GY1 gBI dxY ozX mP2 tGP
nD2 hFL oIY rwP qCX pKM HNU dHW qKT xHX tBH cvE fU3 aMR czA izZ dvR IWZ fyM oMV
gCZ gzL euI BFV ovU kVY mwR wIM eAW kI2 iuH ju3 qwH sFX gVX mV3 sNQ syG ruy byV
suU fJZ mCG wyX EJX LNY s23 eSX kCS svJ qMY nHJ kQR bOW aCQ lUZ eGL gHM pDU fLQ
iK1 FQS cHY dC2 ivB rGM lIL lKO lY2 uJM mNZ kGZ nIK lF3 lBQ rIV BPZ qRV vAK muW
tJY rF1 pBL rQT jGK OZ2 fFY kAF dQZ pFN iE2 iAJ gyS mH1 rJ3 mSY jRZ rES jzP mKX
qEZ jN2 mAD zU2 mMU kvN eMP cS3 mBE pGX gG2 bFU qx2 qzI iNT pW3 HZ3 oT2 pxJ gxN
fPW CT1 gvT iQW GNW