PBL Exercise 1: Consider a "bullet rocket" fired upwards with an initial speed $s_0$. Is it possible for the rocket to escape from the Earth if $s_0$ is large enough, or will it always fall back to Earth? Find the escape speed $s_0$ if possible.

Yes. It is possible for the rocket to escape from the Earth.

http://www.astronomynotes.com/gravappl/escvel.gif
To find escape velocity:

1. **Given Formula**

\[
\frac{GM}{r^2} = \frac{d}{dr} \frac{1}{2} S^2
\]

By variable separable:

\[
\int_{R}^{r+R} \left( \frac{GM}{r^2} \right) dr = \left[ \frac{1}{2} S^2 \right]_{S_0}^{S_i}
\]

\[
- \left[ -GM R^{-1} \right] R = \frac{1}{2} S_0^2 - \frac{1}{2} S_i^2
\]

\[
- \left[ -GM (R+r)^{-1} - GM R^{-1} \right] = \frac{1}{2} \left[ S_0^2 - S_i^2 \right]
\]

\[
\frac{GM}{R+r} + \frac{GM}{R} = \frac{1}{2} \left[ S_0^2 - S_i^2 \right]
\]

For an object to escape Earth

\[ r = +\infty \quad , \quad S_i = 7.0 \]

To find the exact escape velocity

\[ r = +\infty \quad , \quad S_i \text{ approaches } 0 \]

\[
\therefore \lim_{r \to +\infty} \left( \frac{GM}{R+r} + \frac{GM}{R} \right) = \lim_{S_i \to 0} \frac{1}{2} \left[ S_0^2 - S_i^2 \right]
\]

\[
\frac{GM}{R} = \frac{1}{2} S_0^2
\]

\[
S_0 = \sqrt{\frac{2GM}{R}}
\]

\[
= \sqrt{2gR}
\]

*Note: Where \( g = 9.8 \text{ m/s}^2 \) \( R = 6371 \text{ KM (Earth Radius)} \)*

\[ S_0 = 11,174 \text{ Km/s} \]
PBL Exercise 2: Consider a satellite launched from a rocket in a direction tangent to the Earth's surface at a height of 100km.

i) What should the initial speed of the satellite be for it to go into a circular orbit around the Earth?

\[
\mathbf{2.1} \quad V_0 = \sqrt{g \cdot r}
\]

\[
g = 9.8 \text{ m/s}^2 \quad r = 6371 + 100 = 6471 \text{ km (Distance between the two objects)}
\]

\[
V_0 = \sqrt{(9.8)(6471 \times 10^3)}
\]

\[
= 7963 \text{ m/s}
\]

\[
= 7963 \text{ km/s}
\]
ii) What is the least initial speed it can have to remain in an orbit around the Earth, i.e. not fall back to Earth?

By using Elliptical orbit,

http://www.isset.org/nasa/tss/aerospacescholars.org/scholars/earthstationmoon/Unit2/Unit2_1_files/orbitdefs_lg.gif
ii) \[ V_0 = \sqrt{\frac{2GM}{r} - \frac{GM}{a}} \]

\[ a = \frac{(6371 \times 2) + 100}{2} = 6421 \text{ Km} \]

\[ r = 6371 + 100 \]

\[ = 6471 \text{ Km} \]

\[ = 6471000 \text{ m} \]

\[ G = 6.674 \times 10^{-11} \text{ m}^3 \text{ Kg}^{-1} \text{ s}^{-2} \text{ (Newton Constant)} \]

\[ M = 5.973 \times 10^{24} \text{ Kg} \]

\[ \leq V_0 = \sqrt{\frac{2(6.674 \times 10^{-11})(5.973 \times 10^{24})}{6.471000}} \]

\[ = 7.82 \text{ Km/s} \]

iii) What is the greatest initial speed it can have to remain in an orbit around the Earth

\[ V_{\text{max}} = \lim_{a \to +\infty} \sqrt{\frac{2GM}{r} - \frac{GM}{a}} \]

\[ = \sqrt{\frac{2GM}{r}} \]

\[ r = 6471 \text{ Km} \]

\[ V_{\text{max}} = \sqrt{\frac{2(6.674 \times 10^{-11})(5.973 \times 10^{24})}{6471000}} \]

\[ = 11094 \text{ m/s} \]

\[ = 11.1 \text{ km/s} \]

**Conclusion**