SYSTEMS OF POLYNOMIAL INEQUALITIES OVER QUADRATIC MAPS: COMPLEXITY, ALGORITHMS, APPLICATIONS

DATE & TIME of SEMINAR: Tuesday, 25 October 2005 at 3:00pm – 4:00pm
VENUE: NIE 5-01-04 (Executive Seminar Room)
SPEAKER: Dr Dmitrii V. Pasechnik (Tilburg University, Netherlands)

Abstract:

A semialgebraic set (i.e. the union of solutions of a finite number of systems of polynomial inequalities) $S$ is said to be defined over a map $Q$ if $S$ is given by a formula of the form $F(Q(X))$, where $Q: \mathbb{R}^n \rightarrow \mathbb{R}^k$ is a polynomial map and $F(Y)$ is a quantifier-free Boolean formula with polynomial inequalities (with polynomials of degree at most $d$ belonging to a subset of $\mathbb{R}[Y]$ of size $s$) as atoms.

We concentrate on a nontrivial and important for applications case when $Q$ is quadratic. We show that the behaviour of $S$ differs rather drastically from the behaviour of a general $n$-variate semialgebraic set given by degree $d$ polynomials. For instance, the sum of the Betti numbers of $S$ is bounded by $(sdn)^{O(k)}$, and a similar bound holds for the complexity of sampling in $S$ (i.e. computing representatives of the connected components of $S$).

Important applications of our results are, for instance:

1) A polynomial-time algorithm for the extended trust-region problem (a problem of minimising a quadratic function subject to a fixed number $k$ of quadratic constraints)
2) The best known complexity bound for solving semidefinite programming problems exactly.

About the speaker:

Dr. Dmitrii (Dima) Pasechnik is currently a research fellow at the Econometrics and OR Department of Tilburg University. He holds a PhD (with Distinction) in Mathematics from the University of Western Australia.

He has published papers on optimisation, computational complexity, combinatorics, algebra, and scientific computing (computational algebra, geometry and logic).

ALL ARE WELCOME!