Playing the Market is Fun: 
Modeling Derivatives Prices by DOLPHIN
NTU Seminar 15 July 2009 Singapore

Koh Hock Lye
River Engineering and Urban Drainage Research Centre REDAC
School of Mathematical Sciences,
Universiti Sains Malaysia, 11800 USM, Pulau Pinang, Malaysia
E-mail: hlkoh@cs.usm.my
Playing the Market is Fun: An Ecologist Field Day, A Mathematician Haven

Koh Hock Lye
Universiti Sains Malaysia, 11800 Penang, Malaysia
Real Durian: SGD 3 per Kg
Durian Derivative, in a Hamper: SGD 300 per Kg
CDS Credit Default Swap is a hamper with expired contents.
Mutually Supportive Relationship

Unlike Modern Banking

AIG vs Egret
Bats Feed each other during times of scarcity, On a person to person Basis.
Good Times Do Not Last Forever

Bad Luck
**KILL (Bubbles)**

- 33.6°

**Chill (Recession)**

- 19.8°

**Thrill (Boom)**

- 24.8°

**POULATION**

**DAYS**
Mussel releases fish; Fish becomes host to larvae to be dispersed by the fish over long distances

Beneficial to both fish and mussel
We Live in Grave Uncertainty
After an earthquake, a tsunami may follow. Move quickly to higher ground.
26 December 2004 tsunami simulation by TUNA
Learn to Climb tree

Kuala Muda
Journal Publications and Funding
All presented papers in SCSTW3 will be published in a proceeding. Selected papers will be published in an ISI journal or an academic book.

Themes
Themes of the workshop include: scientific, computational, technical and engineering aspects of tsunami, as well as social-cultural-economic implications and dimensions. Our primary goal is the development of tsunami resilient communities worldwide.

Topics to be discussed and presented include:
- Engineering design and construction for tsunami impact reduction;
- Coastal zone management and mitigation plans;
- Effective use of tsunami warning systems and mitigation;
- Mechanism of earthquakes and tsunami and their prediction;
- Numerical simulations and physical modeling of tsunami evolutions;
- Tsunami community preparedness;
- Tsunami risk reduction;
- Tsunami rescue operations;
- Tsunami risk mapping and evacuation routes;
- Roles of NGOs;
- Roles of local governments and councils;
- Education towards tsunami resilience;
- Human resource program for tsunami education and outreach;
- Restoration program and city planning after earthquakes and tsunami;
- Role of coastal vegetation and their recovery after tsunami;
- Related subjects such as storm surge, typhoons and coastal flooding etc.

Registration, Financial Assistance and Submission Guidelines
Further information on registration, financial assistance as well as abstract and full paper submission is available on the conference website.

Contact Persons:
Prof Koh Hock Lye        hlkoh@cs.usm.my
Dr Teh Su Yean           su_yean@hotmail.com
                      syteh@usm.my
Fax:                      604-6570910
Address:                School of Mathematical Sciences
                        Universiti Sains Malaysia
                        11800 USM Penang Malaysia

Organized by:

In conjunction with:

Accommodation
1. Hotel Vistana Penang
   Website: https://www.vistanahotels.com/pg/index.html
   http://www.penang-hotels.com/vistana/

2. B-Suite
   Website: http://www.b-suite.com.my/

3. Hotel Equatorial Penang
   Website: www.equatorial.com/pen/

Second Announcement
SCSTW3: South China Sea Tsunami Workshop 3
03-05 November 2009 Eureka Complex,
Minden Main Campus,
Universiti Sains Malaysia, Penang, Malaysia

Website: http://math.usm.my/scstw3/ or http://trip.webbytes.com.my/scstw/

Following the success of SCSTW1 (Taiwan 2007) and SCSTW2 (Shanghai 2008), a consensus has been reached to organize SCSTW3 from 03 to 05 November 2009 in Universiti Sains Malaysia (USM), Eureka Complex, Minden Main Campus, in collaboration with Cornell University USA and Syiah Kuala University Aceh, Indonesia. Other co-organizers include REDAC, School of Mathematical Sciences USM, Malaysian Meteorological Department, Academy of Sciences Malaysia, National Oceanography Directorate, Mercy Malaysia, Grid@USM, TUNA@USM and MANHAM@USM. Consistent with past traditions of SCSTW, the host will provide limited funding to support this workshop, including partial financial support for selected paper presenters to cover travel and hotel. For those who register before 15 September 2009, registration fees will be waived. After that a registration fee of RM 700 or USD 200 per person will be charged. A study trip will be organized on 04 November to conduct onsite visit to Penang beaches impacted by the 26 December 2004 tsunami.

Important Dates (Updated)
First Announcement sent: 15 April 2009
Second Announcement sent: 15 May 2009
Dateline for Abstract Submission: 30 June 2009
Acceptance of Abstract: 15 July 2009
Dateline for Free Registration: 15 September 2009
Dateline for Financial Assistance: 15 July 2009
Offer of Financial Assistance: 30 August 2009
Final Announcement: 15 October 2009
Dateline for Full Paper: 30 October 2009
Workshop: 3-5 November 2009
Tsunami Resilient Communities
At Risk Must be Able To:

1. Identify hazard zones and investments,
2. Develop inundation maps,
3. Disseminate evacuation maps;
4. Turn vague concerns, abstract issues
   To clarified, concrete action plans;
5. Evacuate Timely during a tsunami.
6. Plan for an orderly retirement before a financial tsunami kills you. Easier said than done.
Percentage change in median sales price of existing homes from one year earlier
USA Showing decline since 2005

Source: National Association of Realtors / Haver Analytics
Percentage of loans delinquency or in the foreclosure process, USA showing increasing trend since 2006
S&P 500 financials-sector index (daily close)
Showing constant decline since 2007
KLCI and PGAF indices (Silly?)

Performance of PUBLIC AGGRESSIVE GROWTH FUND from 25-Nov-98 to 25-Nov-08

PUBLIC AGGRESSIVE GROWTH FUND : Total Returns from 25-Nov-98 To 25-Nov-08=102.75%
Kuala Lumpur Composite Index : Total Returns from 25-Nov-98 To 25-Nov-08=71.30%
DOLPHIN Simulation of KLCI

Graph showing the simulation of KLCI over time (month) with two indices represented: KLCI and PAGF.
Barnacle: Resource-Limited Growth

The Great Recession of the 21 Century
Barings Bank of Britain

- Oldest bank in Britain;
- Bankrupt in 1995;
- Lost 827 million pounds in this debacle;
- Caused by a series of bad gambles taken by a single rogue trader;
- Nick Leeson took large unauthorized trading positions in futures and options;
Enron

- Formed in 1985 by merging Houston Natural Gas and InterNorth;
- Created artificial earnings by dubious methods;
- Inadequate risk adjusted capital;
- Massive internal fraud and;
- External collusion.
Long Term Capital Management

• Long Term Capital Management LTCM was founded in 1994 as a hedge fund;
• High profile quants and star traders on board including 1997 Nobel Economics laureates Myrin Scholes and Robert Merton as partner;
• A main strategy was to use mathematical models to extract arbitrage opportunities in security markets, involving risky exposures;
Long Term Capital Management

• With equity of USD 5 billion, LTCM borrowed over USD 125 billion in 1998;
• Initially it earned returns of 40 % in the first two years, but subsequently reduced sharply to 17 % in the third year, suggesting the non-sustainability of this trading strategy;
• In August 1998, the Russian debt moratorium caused a spike in volatility and a flight to quality, a trading environment highly detrimental to LTCM.
Long Term Capital Management

• The convergence play employed by LTCM turned into divergence in yield spread, causing great loss;

• Federal Reserve Bank of New York organized a rescue package, on behalf of a consortium of 14 banks, in exchange for 90% of LTCM equity with an injection of USD 3.5 billion

• The validity of the BS model was questioned. The model failed during a crisis.
# Major financial crisis since 1980

<table>
<thead>
<tr>
<th>Year</th>
<th>Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mid 1980s</td>
<td>Latin debt crisis</td>
</tr>
<tr>
<td>Late 1980s</td>
<td>savings and loan crisis</td>
</tr>
<tr>
<td>1987</td>
<td>Stock market crash</td>
</tr>
<tr>
<td>1990s</td>
<td>Extensive Japanese asset bubbles</td>
</tr>
<tr>
<td>1994</td>
<td>Mexican peso crisis, Latin American crisis</td>
</tr>
<tr>
<td>1997</td>
<td>Asian crisis</td>
</tr>
<tr>
<td>1998</td>
<td>Russian default crisis</td>
</tr>
<tr>
<td>1999</td>
<td>Brazilian crisis</td>
</tr>
<tr>
<td>2000</td>
<td>US technology stock crash</td>
</tr>
<tr>
<td>2001</td>
<td>911 World Trade Center crash</td>
</tr>
<tr>
<td>2009</td>
<td>Global Financial Crisis of the Century</td>
</tr>
</tbody>
</table>
Basel Committee on Banking Supervision

- **Basel I**
  - requirement for a bank to maintain a mandatory minimum capital of 8%;

- **Basel II**
  - minimum capital requirements;
  - supervisory review process and;
  - market discipline.

- The time has arrived for a new Basel III
The Great Depression of 1929

• 1929 Black Tuesday DJI index dropped 20% in one day; Due to **stock bubble**; No regulations;
• Index dropped from 350 to 40 in one year; creating Hoover Ville Shanty Towns all over USA;
• March 2008: Henry Paulson declared that USA economy was sound; → **real estate bubble** bust;
• But by August 2008, **Fannie and Freddie** failed;
• By the Black 17 September 2008, Wall ST collapsed; → Initiated by Housing bubble;
• Lehman Brothers, Goldman Sachs, Morgan Stanley, also failed; **Iceland economy collapsed** too;
The Great Depression of 1929 (cont.)

- Easy credit expansion lead to speculations;
- Low interest rates, liar market, MBS, CDS, from 2000 to 2007;
- US is Living beyond their means;
- Thrift, frugality, hard work thrown out;
- Nixon decoupled the USD from gold 1971;
- De Regulation from 1981 by Reagan: market based economy;
- 1987 collapse of S/L;
Bernard Madoff, Monster
And the people who enabled him.
BY STEVE FISHMAN
The Road Ahead?
The Great Depression of 1929 (cont.)

• Concentration of wealth, gap between rich and poor continue to widen;

• **FDR Roosevelt New Deal**: SEC, FDIC, setup; dam and road etc built to create employments; program failed;

• Economy only recover after WW2 in 1945+;

• **Alan Greenspan and USD collapse?**

• **Barack Obama New Deal ?;**

• **End of US economic dominance?**

• **New Economic Order?**
Option as a Financial Instrument

- Option
  - Call Option
  - Put Option
Introduction: Diffusion in Ecology

- Diffusion models have been extensively used in ecology and physics research;
- A basic and popular approach is to begin with random walks;
- In one dimension, the random walk concept perceives an individual moving a short distance $\gamma$ to either the right or the left in a short time $\tau$ in a completely random manner with even probability $0.5$. 
Binomial or Bernoulli distribution

• The **probability** $p(r, n)$ that a particle would arrive at a location $r$ steps to the right after a total of $n$ steps

$$p(r, n) = \left( \frac{1}{2} \right)^n \frac{n!}{((n + r)/2)!((n - r)/2)!}$$

• When $n$ approaches infinity this binomial distribution converges to the Gaussian distribution

$$\lim_{{n \to \infty}} p(r, n) = \left( \frac{2}{\pi n} \right)^{1/2} \exp\left( -\frac{r^2}{2n} \right)$$
Binomial or Bernoulli distribution

- The probability distribution of continuous random walk is the **normal distribution** often encountered in probability theory and financial models based upon the BS world.

- Set \( x = r \gamma \) and \( t = n \tau \) and regard \((x,t)\) as continuous variables.

\[
p(x,t) = \frac{1}{2(\pi Dt)^{1/2}} \exp\left( -\frac{x^2}{4Dt} \right)
\]

- The **normal distribution** may be verified to satisfy the basic **diffusion model**

\[
\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left( D \frac{\partial C}{\partial x} \right)
\]
Models are Models

• Models are not a good substitute for knowledge;

• Nor replacement for sources of information;

• Fundamental knowledge of market behavior, prudent management and deep experience are required to drive model input and to interpret model output;

• Do not put the blame on models, in as much as do not put models on the altar.
Option Mathematical Models

- Numerical Black Scholes Models;
- Analytical Black Scholes Models;
- Binomial Tree;
- Monte Carlo.
Black Scholes PDE

\[
\frac{\partial f}{\partial t} + (r + r_m - q)S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf
\]

• IC for European Call: at \( t = T \), \( f = \max(S - K, 0) \)
• IC for European Put: at \( t = T \), \( f = \max(K - S, 0) \)
• BC: At small \( S_{\text{min}} \) set \( f = 0 \) for call
• BC: At large \( S_{\text{max}} \) set

\[
f(S_{\text{max}}, t) = S_{\text{max}} e^{-q(T-t)} - Ke^{-r(T-t)} \approx (S - K)
\]

for call
Explicit Finite Difference Scheme

- To solve the BS PDE numerically by EFD
- Divide the price domain into $2M+1$ points and the time domain into $N$ points
Explicit Finite Difference Equations

- Let \( u_{i,j} = u(X_i, t_j) \), \( X_i = i\Delta X \), \( t_j = j\Delta t \), \( i = -M, \ldots, -1, 0, 1, 2, \ldots M \), \( j = 1, 2, \ldots N \).

- Set \( \Delta t = \frac{T}{N} \), \( S(j) = S* \exp(j\Delta X) \), \( \Delta X = \sigma \sqrt{\frac{3\Delta t}{2}} \).

- Define \( \alpha = \frac{\Delta t}{(\Delta X)^2} \), \( \beta = \mu \frac{\Delta t}{\Delta X} \).

- Let \( a = \frac{1}{2}(\sigma^2 \alpha + \beta) \), \( b = 1 - \sigma^2 \alpha \), \( c = \frac{1}{2}(\sigma^2 \alpha - \beta) \), \( a + b + c = 1 \).

- Then we have \( u_{i,j} = au_{i+1,j+1} + bu_{i,j+1} + cu_{i-1,j+1} \) and since \( f_{i,j} = u_{i,j} e^{-r(T-t_j)} \), we have \( f_{i,j} = e^{-r\Delta t} (af_{i+1,j+1} + bf_{i,j+1} + cf_{i-1,j+1}) \).
Black Scholes Analytical

\[ C = S_0 N(d_1) - K e^{-rT} N(d_2) \]

\[ P = K e^{-rT} N(-d_2) - S_0 N(-d_1) \]

where

\[
\begin{align*}
    d_1 &= \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma \sqrt{T}} \\
    d_2 &= \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma \sqrt{T}} = d_1 - \sigma \sqrt{T}
\end{align*}
\]
Black Scholes Analytical

\[ C = S_0 N(d_1) - Ke^{-rT} N(d_2) \]

\[ P = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \]

where

\[ d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma \sqrt{T}} \]

\[ d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma \sqrt{T}} = d_1 - \sigma \sqrt{T} \]
Binomial Tree

The diagram illustrates a binomial tree with levels indexed by $i = 0, 1, 2, 3, 4, 5$. The tree branches out with options $S$, $Sd^1$, $Sd^2$, $Sd^3$, $Sd^4$, $Sd^5$ at each level. The branches are labeled with probabilities $p$ and $1-p$ to move up or down, respectively. The terminal nodes at level 5 are marked as $S_5, S_4, S_3, S_2, S_1$. Each node represents a possible state or outcome at a given level in the binomial process.
Monte Carlo Method

- Monte Carlo is similar to Random Walk;
- Generate paths of random walk;
- Calculate the average;
- Geometric Brownian Motion (GBM);
- Stochastic Differential Equation SDE:

\[ dS = \mu S dt + \sigma S dz ; \mu = r - q + r_m \]
KLCl and PAGF Indices

Performance of PUBLIC AGGRESSIVE GROWTH FUND from 25-Nov-98 to 25-Nov-08

PUBLIC AGGRESSIVE GROWTH FUND: Total Returns from 25-Nov-98 to 25-Nov-08 = 102.75%  
Kuala Lumpur Composite Index: Total Returns from 25-Nov-98 to 25-Nov-08 = 71.30%
KLCI and PAGF Indices (cont.)

• The traditional BS models (with $r_m = 0$) would not reproduce the sharp inverted V shape;

• To reproduce the sharp inverted V shape, fitting the value of market return $r_m$ to the modified MBS model for KLCI and PAGF.
Kuala Lumpur Composite Index (KLCI)

- We fit the value of $r_m = 0.6\ (60\ \%)\ pa$ (per year) to the modified MBS model;
- We reproduce the shape of KLCI reasonably well, for the period beginning at 81 in June 2006, and peaking at 195 on December 2007;
- Subsequently, KLCI lost at an irrational market return of $1.0\ (100\%)\ pa$, from the peak.
Public Aggressive Growth Fund (PAGF)

• We fit the value of \( r_m = 0.8 \ (80\%) \text{ pa (per year)} \) to the modified MBS model;

• We reproduce the overall shape of PAGF, for the period beginning at 81 in June 2006, and peaking at 260 on December 2007;

• Subsequently, PAGF lost at an irrational market return of 120% pa from the peak.
KLCI relative Index vs. Volatility

- At the market return rate of 0.6 pa in the drift term, variation of volatility within the range of 0.2 to 1.5 pa has no impact on the KLCI index.

<table>
<thead>
<tr>
<th>σ</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>1.0</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>KLCI</td>
<td>195</td>
<td>196</td>
<td>195</td>
<td>196</td>
<td>196</td>
<td>193</td>
<td>196</td>
</tr>
</tbody>
</table>
DOLPHIN Version 1.1

By Koh Hock Lye
DOLPHIN Interface
Conclusion

• Mathematics education should incorporate the development of human resources to enable a rational and scientific valuation of derivatives;
• We need to develop a vibrant and sustainable derivatives market;
• Software such as DOLPHIN can provide an easy introduction into this field;
• To promote cross cultural interaction.