Tiling space by translates of a convex body, with multiplicity

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**Date:** 15 September 2011 (Thursday)
**Time:** 4.30pm – 5.30pm
**Venue:** SPMS-LT3, SPMS-03-02
School of Physical and Mathematical Sciences

We review some of the history of tilings of space by translations, and generalize this theory to include the problem of covering \( \mathbb{R}^d \) by overlapping translates of a convex body \( P \), such that almost every point of \( \mathbb{R}^d \) is covered exactly \( k \) times, for a fixed integer \( k \). Such a covering of Euclidean space by translations is called a \( k \)-tiling. The traditional investigation of tilings (i.e. 1-tilings in this context) by translations began with the work of Fedorov and Minkowski. Here we extend the investigations of Fedorov and Minkowski to \( k \) tilings by proving that if a convex body \( k \)-tiles \( \mathbb{R}^d \) by translations, then it is centrally symmetric, and its facets are also centrally symmetric. The methods are very new, and they allow us to prove analogues of Minkowski's conditions for 1-tiling polytopes. Conversely, in the case that \( P \) is a rational polytope, we also prove that if \( P \) is centrally symmetric and has centrally symmetric facets, then \( P \) must \( k \)-tile \( \mathbb{R}^d \) for some positive integer \( k \).

This is joint work with Nick Gravin and Dmitry Shiryaev.

**Speaker Biography**
Prof. Sinai Robins completed his Ph.D. work at UCLA, after which he held the prestigious National Science Postdoctoral Fellowship at UCSD, and at the Mathematical Sciences Research Institute in Berkeley, in the United States. He has also consulted for the Institute for Defense Analysis, has held a tenured professorship at Temple University before joining NTU in 2008, and was a visiting professor at the Technion, in Israel. He has received the NSA Young Investigator Award from 2000-2003. He has also supervised a number of Ph.D. and Masters students and enjoys mentoring undergraduate students at NTU. His research interests include Discrete and Computational Geometry, Combinatorics, and Number Theory. Prof. Robins enjoys doing Mathematics at the local coffee shops.

**Host:** Division of Mathematical Sciences, School of Physical and Mathematical Sciences

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