Some Improvements of Non-Blackbox Cube Attacks

Yosuke Todo
NTT Secure Platform Laboratories

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Today’s Talk

1. Cube Attacks on Non-Blackbox Polynomials.
   - Proposed at CRYPTO 2017.
   - New generic tools for the cube attack.

2. Improvement 1.
   - Longer distinguisher is found when inactive bits are 0.
   - In detail, ePrint/2017/306.

3. Improvement 2.
   - Reduce the time complexity by exploiting low degree property of superpoly.
   - In detail, ePrint/2017/1063.
Cube Attacks on Non-Blackbox Polynomials
(from CRYPTO2017)

Yosuke Todo, NTT Secure Platform Laboratories, Japan
Takanori Isobe, University of Hyogo, Japan
Yonglin Hao, Tsinghua University, China
Willi Meier, FHNW, Switzerland
Stream ciphers

• Consists of two parts
  - Key initialization.
    • Secret key and public IV are loaded to the internal state.
    • Execute the update function iteratively w/o output of key-stream sequence.
  - Key-stream generation.
    • Update function outputs key-stream sequence.
Example of Trivium: Internal State

state size = 288 bits
Example of Trivium: Key initialization

- 80-bit secret key
- 80-bit initialization vector
- State size = 288 bits
- Initialization = 1152 rounds
Example of Trivium: Output key stream

1 update function outputs 1-bit key stream.
Stream ciphers

- $\vec{x}$ is n-bit secret variable.
- $\vec{v}$ is m-bit public variable.
- $z$ is the first bit of the key stream.

$z = f(\vec{x}, \vec{v}) = \bigoplus a_{\vec{u}}^f \cdot \vec{x}^{\vec{u}} \cdot \vec{v}^{\vec{v}}$

$\vec{u} \in \mathbb{F}_2^{n+m}$

ex) $z = x_1 x_2 \oplus x_1 v_1 \oplus v_2 v_3$
Idea of the cube attack [DS09]

\[ t_I = \nu_{i_1} \times \cdots \times \nu_{i_{|I|}} \]

- Let \( I = \{i_1, \ldots, i_{|I|}\} \) be the indices of active bits.

- Let \( C_I \) be a set of \( 2^{|I|} \) values where \( \nu_i \ (i \in I) \) is active.

\[
\begin{align*}
\vec{x} &= (x_1, \ldots, x_n) \\
\vec{v} &= (v_1, \ldots, v_m)
\end{align*}
\]

\[
z = f(\vec{x}, \vec{v}) = t_I \cdot p_I(\vec{x}, \vec{v}) + q_I(\vec{x}, \vec{v})
\]

\[
\bigoplus_{v \in C_I} z = p_I(\vec{x}, \vec{v})
\]

Attackers recover secret variable \( \vec{x} \) by analyzing \( p_I(\vec{x}, \vec{v}) \).
Concrete example

\[ f(v_1, v_2, v_3, x_1, x_2) \]
\[ = v_1 v_2 v_3 + v_1 v_2 x_1 + v_2 x_1 x_2 + v_1 v_2 + v_2 + v_3 x_2 + x_2 + 1 \]
\[ = v_1 v_2(v_3 + x_1 + 1) + (v_2 x_1 x_2 + v_3 x_2 + v_2 + x_2 + 1) \]

\[
\begin{align*}
  t_I &= v_1 v_2 \\
  p_I(\vec{x}) &= v_3 + x_1 + 1 \\
  q_I(\vec{x}) &= v_2 x_1 x_2 + v_3 x_2 + v_2 + x_2 + 1
\end{align*}
\]

\[ \bigoplus_{(v_1, v_2) \in \{0,1\}^2} f(\vec{v}, \vec{x}) = v_3 + x_1 + 1 \]
Unfortunately...

Let $I = \{i_1, ..., i_{|I|}\}$ be the indices of active bits.

Let $C_I$ be a set of $2^{|I|}$ values where $v_i$ ($i \in I$) is active.

We cannot decompose $f(\bar{x}, \bar{v})$ because real stream cipher is complicated.
Experimental balckbox analysis

• How to recover $p_I(\hat{x}, \hat{v})$.
  1. Assume that $p_I$ is linear function.
  2. Randomly choose $\hat{x}$.
     iteratively compute $\bigoplus_{\hat{v} \in C_I} f(\hat{x}, \hat{v}) = p_I(\hat{x}, \hat{v})$.
  3. Execute linearly test on many $\hat{x}$.
     Recover $p_I$ under the assumption that it’s linear.

• Drawback
  - The cube size is limited in the range of experimental, e.g., $|C_I| \leq 40$. 
Motivation

Experimental cube attack

- Iterate linearly test experimentally.
- Recover the ANF of superpoly in real.

Theoretical cube attack

- Analyze the structure of superpoly.
- Evaluate the ub to recover its ANF.

We use the *division property* as a tool to analyze the structure of the superpoly.

Stream ciphers

\[ \vec{x} = (x_1, \ldots, x_n) \quad \vec{v} = (v_1, \ldots, v_m) \]

\[ z = f(\vec{x}, \vec{v}) \]
Division Property

If there is NOT division trail \( \overrightarrow{k_0} \)\( \circ \cdots \circ F_1 \rightarrow \overrightarrow{1} \),
the output of the Boolean function \( f \) is balanced.

\[ \overline{\overrightarrow{k_0}} = t_I. \]
How to analyze division trails?

• Programming from scratch.
  - Depth/Breadth First Search.

• CP-based approaches.
  - Mixed Integer Linear Programming.
  - SAT solver.
  - Constraint Programming.
Zero-sum distinguisher

\[ t_I = v_{i_1} \times \cdots \times v_{i_{|I|}} \]

\[ \vec{x} = (x_1, \ldots, x_n), \quad \vec{v} = (v_1, \ldots, v_m) \]

Division property

\[ (\vec{0}, \vec{k}), \quad \vec{v}^k = t_I \]

No division trail.

\[ 1 \]

\[ \oplus_{v \in C_I} f(\vec{x}, \vec{v}) = 0 \]

Zero-sum distinguisher is trivially application.
How to recover the ANF.

• The role of division property.

Integral characteristic search tool. \rightarrow ANF coefficients search tool.

• We revisit what the division property can do.
What division property can do

- Assuming there is NOT trail

\[ \overrightarrow{k} \xrightarrow{\frac{f(\overrightarrow{x})}{}} 1, \]

\[ \bigoplus_{C_1} f(\overrightarrow{x}) = p(\overrightarrow{x}) = \bigoplus_{\overrightarrow{u} \in \mathbb{F}_2^n | \overrightarrow{u} \geq \overrightarrow{k}} a_{\overrightarrow{u}}^f \cdot \overrightarrow{x} \overrightarrow{u} \oplus \overrightarrow{k} \]

is always zero for any \( \overrightarrow{x} \).

- In other words,

- \( a_{\overrightarrow{u}}^f \) is always 0 for any \( \overrightarrow{u} \geq \overrightarrow{k} \).

- Division property can be used to analyze ANF coefficients.
Extension to key recovery.

- Assuming there is NOT trail \((\vec{e}_j, \vec{k}) \xrightarrow{\text{f}(\vec{x}, \vec{v})} 1\), \(a_{\vec{u}}^f\) is always 0 for any \(\vec{u} \geq (\vec{e}_j || \vec{k})\).

- Then,

\[
\bigoplus_{C_1} f(\vec{x}, \vec{v}) = p(\vec{x}, \vec{v}) = \bigoplus_{\vec{u} \in \mathbb{F}_{2^{n+m}}^+ | \vec{u} \geq (\vec{0} || \vec{k}_I)} a_{\vec{u}}^f \cdot (\vec{x} || \vec{u}) \vec{u} \oplus \vec{0} || \vec{k}_I
\]

\[
= \bigoplus_{\vec{u} \in \mathbb{F}_{2^{n+m}}^+ | \vec{u} \geq (\vec{0} || \vec{k}_I), u_j = 0} a_{\vec{u}}^f \cdot (\vec{x} || \vec{u}) \vec{u} \oplus \vec{0} || \vec{k}_I.
\]

- The superpoly is independent of \(x_j\) because \(x_j^{u_j} = x_j^0 = 1\).
Summary of division property-based cube

Stream ciphers

\[ x = (x_1, \ldots, x_n) \quad \vec{v} = (v_1, \ldots, v_m) \]

\[ t_I = v_{i_1} \times \cdots \times v_{i_{|I|}} \]

Division property

\[ \left( e_j, k \right), \quad \vec{v}^k = t_I \]

No division trail.

1

By repeating this procedure, we can distinguish which secret-key bits are involved.
Applications.

<table>
<thead>
<tr>
<th>Applications</th>
<th>Previous Best</th>
<th>New Best</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trivium</td>
<td>799</td>
<td>832</td>
</tr>
<tr>
<td>Grain128a</td>
<td>177</td>
<td>183</td>
</tr>
<tr>
<td>ACORN</td>
<td>503</td>
<td>704</td>
</tr>
<tr>
<td><strong>Kreyvium</strong></td>
<td><strong>--</strong></td>
<td><strong>872</strong></td>
</tr>
</tbody>
</table>

※Applications to Kreyvium are explained the full version (ePrint/2017/306)
1st Improvement.
Exploiting constant-0 cubes.
(from ePrint/2017/306)

Yosuke Todo, NTT Secure Platform Laboratories, Japan
Takanori Isobe, University of Hyogo, Japan
Yonglin Hao, Tsinghua University, China
Willi Meier, FHNW, Switzerland
Motivation

We want to fill the gap from other works.

Non-active bits are always 0 in many previous cubes.

\[
\vec{x} = (x_1, \ldots, x_n) \quad \vec{v} = (v_1, \ldots, v_m)
\]

Non-active bits are any value in our cubes.

\[
f(v_1, v_2, v_3, x_1, x_2)
= v_1 v_2 (v_3 + x_1 + v_3 x_2 + 1) + (v_2 x_1 x_2 + v_3 x_2 + v_2 + x_2 + 1)
\]

\[
p(v_3, x_1, x_2) = v_3 + x_1 + v_3 x_2 + 1
\]

\[
p(0, x_1, x_2) = x_1 + 1
\]
Motivation

We want to fill the gap from other works.

Non-active bits are always 0 in many previous cubes.

\[ \overrightarrow{x} = (x_1, \ldots, x_n) \quad \overrightarrow{v} = (v_1, \ldots, v_m) \]

Non-active bits are any value in our cubes.

\[ \overrightarrow{x} = (x_1, \ldots, x_n) \quad \overrightarrow{v} = (v_1, \ldots, v_m) \]

- 0-constant cubes bring more powerful attack generally.
- Liu’s cube (at CRYPTO17) also uses 0-constant cube.

We need a new technique to exploit 0-constant cube with the division property.
Exploiting the constant 0

- Non-cube bits are 0.

\[ \vec{x} = (x_1, \ldots, x_n) \quad \vec{v} = (v_1, \ldots, v_m) \]

- If non-cube bit is fixed to 0, the propagation of the division property is restricted.

\[
\begin{align*}
\nu_1 & \quad \nu'_1 & \quad (0,0) \rightarrow (0,0,0) \\
\nu_2 & \quad \nu'_1 & \quad (1,0) \rightarrow (1,0,0), (0,0,1) \\
\& & \quad \nu'_2 & \quad (0,1) \rightarrow (0,1,0), (0,0,1) \\
\& & \quad \nu'_3 & \quad (1,1) \rightarrow (1,1,0), (0,0,1)
\end{align*}
\]

※Similar technique was already used by Sun et al’s work in the context of the integral distinguisher (ePrint/2016/1101).
Exploiting the constant 0

- Non-cube bits are 0.

- If non-cube bit is fixed to 0, the propagation of the division property is restricted.

\[
\vec{x} = (x_1, \ldots, x_n) \quad \vec{v} = (v_1, \ldots, v_m)
\]

\[v_2 = 0\] impossible propagation

\[
\begin{align*}
\nu_1 & \rightarrow \nu'_1 \quad (0,0) \rightarrow (0,0,0) \\
\nu_2 & \rightarrow \nu'_2 \quad (1,0) \rightarrow (1,0,0), (0,0,1) \\
\& & \rightarrow \nu'_3 \quad (0,1) \rightarrow (0,1,0), (0,0,1) \\
& & \quad (1,1) \rightarrow (1,1,0), (0,0,1)
\end{align*}
\]

※Similar technique was already used by Sun et al’s work in the context of the integral distinguisher (ePrint/2016/1101).
Summary of distinguishing attacks.

<table>
<thead>
<tr>
<th>Applications</th>
<th>rounds</th>
<th>cube size</th>
<th>type</th>
<th>method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trivium</td>
<td>837</td>
<td>37</td>
<td>zero sum</td>
<td>Liu &amp; ours</td>
</tr>
<tr>
<td></td>
<td>838</td>
<td>38</td>
<td>zero sum</td>
<td>ours</td>
</tr>
<tr>
<td></td>
<td>842</td>
<td>37</td>
<td>biased sum</td>
<td>experimental (Liu)</td>
</tr>
<tr>
<td>Kreyvium</td>
<td>872</td>
<td>61</td>
<td>zero sum</td>
<td>Liu &amp; ours</td>
</tr>
<tr>
<td></td>
<td>873</td>
<td>62</td>
<td>zero sum</td>
<td>ours</td>
</tr>
</tbody>
</table>

- We can revisit Meicheng Liu’s result.
- We can improve the zero-sum distinguisher on Trivium and Kreyvium from Liu’s result.
- We haven’t tried experimental approaches.
  - There is the possibility 38-dimensinal cube derives stronger biased sum distinguisher.
## Comparison between Liu’s result

<table>
<thead>
<tr>
<th></th>
<th>Liu’s algorithm</th>
<th>Division property</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complexity</td>
<td>WIN</td>
<td>LOSE</td>
<td>We need to ask for solver’s help to evaluate the division trails.</td>
</tr>
<tr>
<td>Accuracy</td>
<td>LOSE</td>
<td>WIN (w/ improved technique.)</td>
<td>I find some instances that division property is better than Liu’s algorithm.</td>
</tr>
<tr>
<td>Flexibility</td>
<td>LOSE</td>
<td>WIN</td>
<td>Division property is applicable to arbitrary ciphers.</td>
</tr>
</tbody>
</table>

- **Recommendation.**
  - If the solver can stop, division property is better.
  - Otherwise, e.g., the state size is too large, we have to use Liu’s algorithm.
2nd Improvement.
Exploiting Low Degree Property of Superpoly (ePrint/2017/1063)

Qingju Wang, DTU, Denmark
Yonglin Hao, Tsinghua University, China
Yosuke Todo, NTT Secure Platform Laboratories, Japan
Chaoyun Li, KU Leuven, Belgium
Takanori Isobe, University of Hyogo, Japan
Willi Meier, FHNW, Switzerland
Motivation

Experimental cube attack.

• Superpoly is assumed as linear or quadratic.
  – Experimental cube recovers superpoly efficiently by exploiting this low degree property.

Take one step further!!

• We also exploit this low-degree property with the division property.
  – The upper bound of the degree on superpoly is estimated.
  – The time complexity is more reduced.
If the superpoly is low degree,...

- If the degree is at most $d$.
  - We don’t need to evaluate the ANF coefficients whose degree of monomials is more than $d$.
  - The time complexity is reduced from

$$2^{|I| + |J|} \text{ to } 2^{|I|} \times \sum_{i=0}^{d} \left( \begin{array}{c} |J| \\ i \end{array} \right)$$


How degree is evaluated?

\[ t_I = v_{i_1} \times \cdots \times v_{i_{|I|}} \]

\[ \bar{x} = (x_1, \ldots, x_n) \quad \bar{v} = (v_1, \ldots, v_m) \]

Stream ciphers

\[ z = f(\bar{x}, \bar{v}) \]

Division property

\[ (\ell, k), \quad v^\ell_k = t_I \]

No division trail.

\[ \underbrace{1}_{\text{under this condition}} \]

maximize \( \sum_{j \in J} \ell_j \)

This maximum value corresponds the upper bound of the algebraic degree of the superpoly.
Applications and results

| Applications | rounds | cube size | $|J|$ | time          | ref.             |
|--------------|--------|-----------|------|---------------|-----------------|
| Trivium      | 832    | 72        | 5    | $2^{77}$      | crypto17        |
|              | 839    | 78        | 1    | $2^{79}$      | ePrint/2017/1063|
| Kreyvium     | 872    | 85        | 39   | $2^{124}$     | ePrint/2017/306 |
|              | 888    | 102       | 36   | $2^{111.38}$  | ePrint/2017/1063|

- Focus on 888-round attack on Kreyvium.
  - The number of involved secret variables is 36.
  - Previous estimations requires $2^{138}$ complexity.
  - However, since the degree of superpoly is at most 2, we can dramatically reduce the complexity.
Conclusion

• Division property based cube attacks
  - A new generic framework to evaluate the security against cube attacks.
  - It brings best key-recovery attacks against Trivium, Grain128a, ACORN, Kreyvium.

• Further improvements
  - Exploiting constant-0 cube brings more powerful superpoly recovery attacks.
  - Exploiting low degree property of the superpoly reduce the time complexity to recover the superpoly.