Automatic Search of Impossible Differentials and Zero-Correlation Linear Hulls^{*}

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Outline

➢ Introduction

>Automatic Search of Impossible Differentials

Automatic Search of Zero-Correlation Linear Hulls

Discussions and Conclusions

Outline

➢ Introduction

- Impossible differential cryptanalysis (IDC) and zerocorrelation linear cryptanalysis (ZCLC)
- Previous tools for finding impossible differentials (IDs) and their disadvantages
 - Our contributions
- Automatic Search of Impossible Differentials
 Automatic Search of Zero-Correlation Linear Hulls
 Conclusions

--Impossible differential cryptanalysis and zero-correlation linear cryptanalysis

Impossible Differential Cryptanalysis (IDC)

- One of the most popular cryptanalytic tools for block ciphers
- Proposed by Knudsen (1998) to attack DEAL and extended by Biham et al. to analyze IDEA and Skipjack
- Exploits differentials with probability zero to recover keys
- Zero-Correlation Linear Cryptanalysis (ZCLC)
 - The counterpart of IDC in the domain of linear cryptanalysis
 - Exploits zero-correlation linear hulls (ZCLH) to recover keys
 - Has attracted much attention in the last two years (DCC 2012, FSE'12, Asiacrypt'12, SAC'13), and has been proved to be more powerful than IDC in some cases

——Impossible differential cryptanalysis and zero-correlation linear cryptanalysis

➢ Perform a successful IDC and ZCLC

■ Two important factors

• Length of an ID/a ZCLH, number of IDs/ZCLHs

■ Length of an ID/a ZCLH

• The longer the ID/ZCLH is, the better the attack will be.

■ Find more IDs/ZCLHs

• More IDs/ZCLHs we find, more probabilities we can perform a successful attack or improve known attacks.

——Previous tools and their disadvantages

$\succ U$ -method and UID-method

- In Indocrypt 2003, Kim et al. proposed the *U*-method to find impossible differentials for block cipher structures with bijective round functions.
- Extended by Luo et al. (2009), and named as the UID-method
- Based on the miss-in-the-middle approach

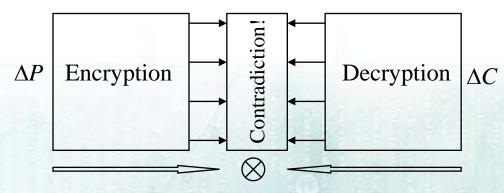


Fig 1. Basic idea of the miss-in-the-middle approach

Employed by the designers of LBlock (ACNS'11), Piccolo (CHES'11) and TWINE (SAC'12)

——Previous tools and their disadvantages

Disadvantages of previous tools

■ Miss-in-the-middle approach limits their power

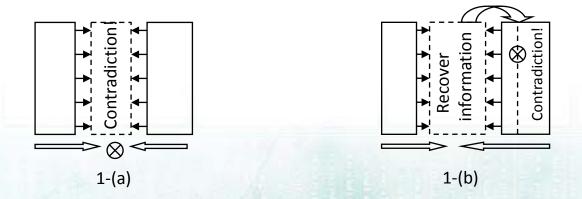


Fig 2. Miss-in-the-middle approach (1-(a)) and IDs with information feedback (1-(b))

Failed to find the longest known IDs of many block ciphers, such as Camellia, MIBS and E2

■ There is a gap between previous tools and ad hoc approaches



- Proposed a new tool to search truncated IDs for word-oriented block ciphers with bijective Sboxes (Indocrypt'12)
 - The *U*-method and the UID-method are specific cases of our tool, and our tool is more powerful than them.
 - Although our tool does not improve the lengths of IDs for existing word-oriented block ciphers, it helps in reducing the gap between previous tools and ad hoc approaches
 - Not only rediscovers the longest truncated IDs of many wordoriented block ciphers known so far, but also finds new results

--Our contributions

Table 1. Summary of new truncated IDs for word-oriented block ciphers.

Block cipher	Word unit	Previous results				In this paper	
		Round	No. of IDs	Method	Round	No. of IDs	New IDs
CLEFIA	Byte	9	72	ad hoc	9	72	0
AES	Byte	4	269,554	ad hoc	4	3,608,100	3,338,546
ARIA	Byte	4	156	ad hoc	4	94,416	94,260
Camellia*	Byte	8	3	ad hoc	8	4	1
E2	Byte	6	/ 1	ad hoc	6	56	55
MIBS	Nibble	8	2	ad hoc	8	8	6
LBlock	Nibble	14	64	U-method	14	80	16
Piccolo	Nibble	7	1	U-method	7	450	449

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- Extend our tool to search IDs for bit-based block ciphers (new)
 Use more properties of Sboxes, besides that they are bijective
- Extend our tool to search ZCLHs for word-oriented block ciphers and bit-based block ciphers (new)



——Our contributions

Table 2. Compare the results of IDs and ZCLHs (one 2.66GHz core, Magma)

Block cipher Re		ts of IDs	Time	Results	of ZCLHs	Time
	round	No		round	No	
ARIA	4	94260	2 weeks	4	58128	days
E2	6	56	Hours	6	41	Hours
MIBS	8	6	(Enumer ate half	8	2	(Enumer ate half
Camellia	8	4	side)	8	8	side)
LBlock	14	80	,	14	80	,
PRESENT-48	6	146	days	5	4200	days
PRESENT-64	6	3776	(from 1 Sbox to 1	6	960	(from 1 Sbox to 1
PRESENT-96	6	40621	Sbox to 1 Sbox)	6	12275	Sbox to 1 Sbox)

Note: PRESENT-like ciphers with block size 48/64/96 are in fact block ciphers EPCBC-48, PRESENT, EPCBC-96 (CANS'11), respectively.

Outline

> Introduction

> Automatic Search of Impossible Differentials ■ Sketch of our tool Difference propagation system ■ Some details of our tool • Word-oriented block cipher • Bit-based block cipher Complexity analysis > Automatic Search of Zero-Correlation Linear Hulls Discussions and Conclusions

- Treat a block cipher as an entirety, describe the propagation of differences in a block cipher as a system of equations —— difference propagation system
- Predict information by solving a difference propagation system iteratively
 - Abort condition: a contradiction is detected or we cannot get new information any longer
- A contradiction is detected => the difference propagation system has no solution => an impossible differential is found

——Algorithm for searching IDs

- 1. Build the difference propagation system of a block cipher;
- 2. for each pair of $(\Delta P, \Delta C)$ we choose do;
- 3. *index*:=true;
- 4. **while** *index* **do**;
- 5. Predict information by solving the difference propagation system;
- 6. **if** *a contradiction is found* **then**;
- 7. *index*:=false; **return** true;
- 8. **elseif** *cannot get any new information* **then**;
- 9. *index*:=false;
- 10. **end if**;
- 11. end while;
- 12. **end for**;

Algorithm 1. Algorithm for searching IDs

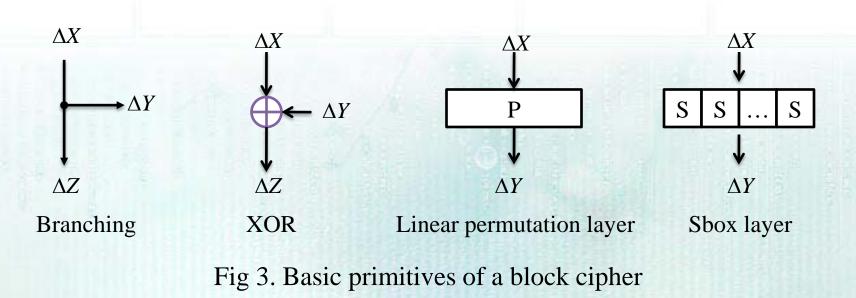
----Algorithm for searching IDs

1.	Build the difference propagation system of a block cipher;
$\overline{2.}$	for each pair of $(\Delta P, \Delta C)$ we choose do ;
3.	<i>index</i> :=true;
4.	while <i>index</i> do;
5.	Predict information by solving the difference propagation system;
6.	if a contradiction is found then;
7.	<i>index</i> :=false; return true;
8.	elseif cannot get any new information then;
9.	<i>index</i> :=false;
10.	end if;
11.	end while;
12.	end for;

Algorithm 1. Algorithm for searching IDs

➢ Build equations for basic primitives

- consider the XOR difference and suppose subkeys are XORed to the state, then the key addition layers can be omitted.
- Four basic primitives are widely used in a block cipher



——Difference propagation system

Suppose $\Delta X = (\Delta x_1, \Delta x_2, ..., \Delta x_n), \Delta Y = (\Delta y_1, \Delta y_2, ..., \Delta y_n)$ and $\Delta Z = (\Delta z_1, \Delta z_2, ..., \Delta z_n)$

Basic primitives	$\begin{array}{c} \Delta X \\ \downarrow \\ \downarrow \\ \Delta Z \end{array} \qquad \Delta Y$	ΔX \downarrow ΔY ΔZ	$ \begin{array}{c} \Delta X \\ \downarrow \\ P \\ \downarrow \\ \Delta Y \end{array} $	$ \begin{array}{c} \Delta X \\ \downarrow \\ S S \dots S \\ \downarrow \\ \Delta Y \end{array} $
Equations	$\Delta x_i \oplus \Delta y_i = 0,$ $\Delta x_i \oplus \Delta z_i = 0$	$\Delta x_i \oplus \Delta y_i \oplus \Delta z_i = 0$	$\Delta y_i \oplus \sum_{j=1}^n P_{i,j} \cdot \Delta x_j = 0$	$\overline{S}(\Delta x_i, \Delta y_i) = 0$
Note	include 2n linear equations	include <i>n</i> linear equations	include <i>n</i> linear equations	include <i>n</i> non- linear equations from <i>n</i> Sboxes

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——Non-linear equation derived from an Sbox

- ➤ Build the difference distribution table (DDT) of an Sbox $F_2^s \rightarrow F_2^t$ (2^s×2^t entries)
 - DDT[i][j]>0 means that the input difference i may propagate to the output difference j, with some probabilities
- Replace each nonzero entry of a DDT with 1
 In algebraic view, the replaced DDT is equivalent to a truth table with 2^{s+t} entries
- Recover the boolean function *f* based on the truth table
 s+t variables represent the input and output difference
 f+1=0 totally catches the difference propagation behavior of this Sbox

——Difference propagation system

- Difference propagation system of a block cipher
 - Combine the equations derived from basic primitives together to build a difference propagation system
 - Choose *r*-round SPN ciphers and Feistel ciphers with SPN round functions as examples

---Difference propagation system

Block ciphers	Difference propagation system	Note
ΔX_{i} \downarrow $S S \dots S$ $\downarrow \Delta Y_{i}$ P \downarrow ΔX_{i+1}	$\begin{cases} \overline{S}(\Delta X_{i,j}, \Delta Y_{i,j}) = 0, \\ \Delta X_{i+1}^T \oplus P \cdot \Delta Y_i^T = 0 \\ (1 \le i \le r, 1 \le j \le n) \end{cases}$	N=(2r+1)n variables rn linear equations rn non-linear equations from Sbox layers
$\Delta X_{i} \qquad \Delta X_{i-1}$ $\downarrow \qquad \qquad$	$\begin{cases} \overline{S}(\Delta X_{i,j}, \Delta Y_{i,j}) = 0, \\ \Delta Z_i^T \oplus P \cdot \Delta Y_i^T = 0, \\ \Delta X_{i-1} \oplus \Delta Z_i \oplus \Delta X_{i+1} = 0 \end{cases}$ $(1 \le i \le r, 1 \le j \le n)$	N=(3r+2)n variables 2rn linear equations rn non-linear equations from Sbox layers

--Difference propagation system and impossible differentials

Theorem. $\Delta P \rightarrow \Delta C$ is an impossible differential, if the difference propagation system initialized by ΔP , ΔC has no solution.

Proof. A block cipher is built by a list of basic primitives and the difference propagation behavior of each primitive is totally described by the corresponding equations. A differential trail in fact defines a solution of the difference propagation system. Thus, the difference propagation system initialized by ΔP , ΔC has no solution implies that there is no differential trails starting from ΔP and ending at ΔC , that is, $\Delta P \rightarrow \Delta C$ is an impossible differential.

Automatic Search of Impossible Differentials ---Solve a difference propagation system

An impossible differential is obtained if we confirm that the corresponding difference

- propagation system has no solution!
- Difficulty and our strategy
 - Solve a nonlinear system is difficult!

Our strategy: predict information by partially solving the system. An impossible differential is confirmed if we find some contradictions during information prediction.

——Algorithm for searching IDs

- Build the difference propagation system of a block cipher; 1.
- for each pair of $(\Delta P, \Delta C)$ we choose do; 2.
- 3. *index*:=true;
- 4. while *index* do;

5.	Predict information by soling the difference propagation system;
6.	if a contradiction is found then;

- if a contradiction is found then;
- *index*:=false; **return** true;
- 8. elseif cannot get any new information then;
- 9. *index*:=false;
- 10. end if;

7.

- end while; 11.
- 12. end for;

Algorithm 1. Algorithm for searching IDs

——Predict information and detect contradictions

- A difference propagation system can be divided into two subsystems — *L* and *NL*
 - *L* includes all linear equations
 - \blacksquare *NL* includes all non-linear equations from the Sbox layers

Example 1. For an *r*-round SPN cipher, we have

$$\begin{cases} \overline{S}(\Delta X_{i,j}, \Delta Y_{i,j}) = 0 \ (1 \le i \le r, 1 \le j \le n), & \longrightarrow \\ \Delta X_{i+1}^T \oplus P \cdot \Delta Y_i^T = 0 \ (1 \le i \le r, 1 \le j \le n) & \longrightarrow \\ \end{cases}$$
 System \mathcal{L}

——Predict information and detect contradictions

\succ Predict information from \mathcal{NL}

Lemma 1. Suppose *S* is a bijective Sbox, *x* is its input difference and *y* is its output difference, then

 $x=0 (\neq 0)$ if and only if $y=0 (\neq 0)$.

Predict information from L

■ If linear system *L* has solutions, we can solve it by Gauss-Elimination algorithm and recover information from the remaining system

Lemma 2. After solving a linear system by Gauss-Elimination algorithm, then 1) If an affine equation with only a variable, i.e., $x \oplus c=0$ (*c* is a constant), is found in the system, then x=0 if c=0 and $x\neq 0$ if $c\neq 0$; 2) If a linear equation with two variables, i.e., $x \oplus y=0$, is found in the system, then $x\neq 0$ if and only if $y\neq 0$.

—Predict information and detect contradictions

Example 2. Suppose we have known that $u=0, x\neq 0$, and the difference propagation system is

 $\overline{S}(u,v) = 0,$ $x \oplus y = 0,$ $x \oplus y \oplus z = 0.$

After one step of information prediction, we have

—Predict information and detect contradictions

Detect contradictions

Proposition 1. For given plaintext difference ΔP and ciphertext difference ΔC , $\Delta P \rightarrow \Delta C$ is impossible if one of the following two situations happens: 1) The linear system \mathcal{L} doesn't have any solution. That is, the rank of its coefficient matrix is not equal to the rank of its augmented matrix; 2) There exists a variable with both zero and penzero values

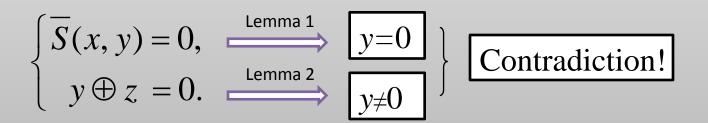
2) There exists a variable with both zero and nonzero values.

> A tiny example of the second type of contradiction

Example 3. Suppose we have known that $x=0, z\neq 0$, and the following equations are included in a difference propagation system

 $\begin{cases} \overline{S}(x, y) = 0, \\ y \oplus z = 0. \end{cases}$

After a step of information prediction, we have



Automatic Search of Impossible Differentials --Drawbacks of Lemma 1

- Lemma 1 is not able to exploit any properties of the Sboxes beyond the fact that they are bijective.
 - May miss some longer IDs if other properties of an Sbox is required in detecting them.
- Suitable for searching truncated IDs of wordoriented block ciphers, where the input/output difference of an Sbox propagates through the block cipher as a unit.
 - Results listed in Tab.1 are obtained by this technique.

Automatic Search of Impossible Differentials --Situations in bit-based block ciphers

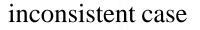
- Situation is changed in bit-based block ciphers
 - Difference propagation system is not wordoriented but bit-oriented, each variable represents a bit.
 - The input/output difference of an Sbox is broken into bits and no longer influences other variables as a unit.
 - •May encounter the situation that partial bits of an Sbox are known while other bits are unknown.
 - •Known bits of an Sbox may help us to retrieve new information of other bits

- Retrieve more information from an Sbox
 A bit of an Sbox can be 0, 1 or ? (3^{t+s} cases)
 For each case
 - Substitute the variables in the non-linear equation with known bits, and then solve it to get new information of the unknown bits.
 - Precompute and store new information in a table
 - In the procedure of information prediction, add new information to the difference propagation system
 - Useful new information
 - The value of a bit
 - A linear relation between unknown bits
 - An inconsistent case

——Situations in bit-based block ciphers

Example 4. Consider the following three cases of PRESENT's Sbox

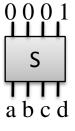
$\begin{array}{c} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\$	
S	
$\begin{array}{c} 1 1 1 1 \\ 0 ? 0 ? \end{array}$	



both ? are zero

000?

S



d=1,b=0,a+c+1=0

The precomputed table is as follows

Input and output difference	New information	
[0001;0?0?]	inconsistent case	
[000a;000b]	a=b=0	
[0001; abcd]	d=1,b=0,a+c+1=0	

——Predict information from an Sboxes

Example 5. Suppose we consider a small version of PRESENT with 16 bits. The plaintext difference is

(0000, 0000, 0000, ****)

while the ciphertext difference is

(0000, 0000, 0000, ****).

Then, we derive that six bits of each Sbox in the second round are zero while other bits are known.

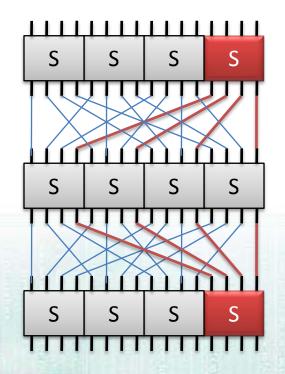


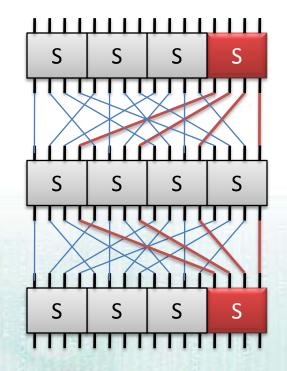
Fig.4. A small version of PRESENT

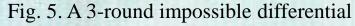
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——Situations in bit-based block ciphers

Example 5. The second case of example 4, that is $\begin{array}{c} 0 & 0 & 0 \\ 1 & 1 & 1 \\ \hline & 5 \\ 0 & 0 & 0 \end{array}$ both ? are zero

implies that differential listed in right side is impossible! This impossible differential can not be detected if we only use the bijective property of PRESENT's Sbox.





Automatic Search of Impossible Differentials --Detect contradictions for bit-based block ciphers and experimental results

Detect contradictions for a bit-based block cipher

Proposition 1'. Besides the condition 1) and 2) given in Proposition 1, Δ*P* → Δ*C* is also impossible if
3) An inconsistent case is found during predicting information from an Sbox.

Experimental results for PRESENT-like ciphers
 After exploiting more properties of the Sboxes beyond that they are bijective, the length of impossible differentials for PRESENT-48/64/96 increase from 2/2/3 round to 6/6/6 round

——Algorithm complexity

Complexity

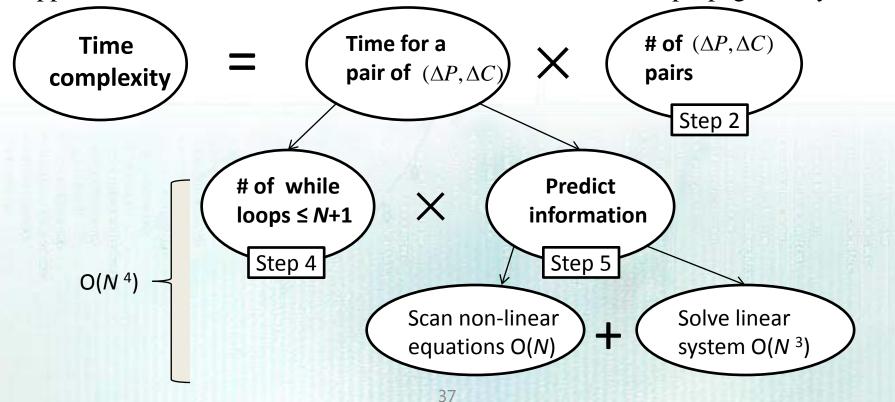
- 1. Build the difference propagation system of a block cipher;
- 2. for each pair of $(\Delta P, \Delta C)$ we choose do;
- 3. *index*:=true;
- 4. **while** *index* **do**;
- 5. Predict information by solving the difference propagation system;
- 6. **if** *a contradiction is found* **then**;
- 7. **return** true; **break**;
- 8. **elseif** *cannot get any new information* **then**;
- 9. *index*:=false;
- 10. **end if**;
- 11. end while;
- 12. **end for**;

Memory complexity: store the difference propagation system and precomputed table.

——Algorithm complexity

- 2. for each pair of $(\Delta P, \Delta C)$ we choose do;
- 4. **while** *index* **do**;
- 5. Predict information by solving the difference propagation system;

Suppose N is the number of variables involved in a difference propagation system.



Outline

> Introduction

- > Automatic Search of Impossible Differentials
- Automatic Search of Zero-Correlation Linear Hulls
 - Sketch of the tool
 - Linear mask propagation system
- Discussions and Conclusions

The theorem of correlation matrix (Daemen, FSE'94)

$$C(\Gamma c \cdot E_k(x), \Gamma p \cdot x) = \sum_U C_U = \sum_{\Gamma p = \alpha_0, \alpha_1, \dots, \alpha_r = \Gamma c} \prod_{i=1}^r C(\alpha_i^t \cdot f_i(x), \alpha_{i-1}^t \cdot x)$$

 $\blacksquare \Gamma p \to \Gamma c \text{ is a ZCLH if}$

• The sum of correlations of all linear trails is zero.

• All linear trails are unbiased. $(\sqrt{})$

> In key-alternating block ciphers

$$C(\Gamma c \cdot E_k(x), \Gamma p \cdot x) = \sum_U (-1)^{d_k} |C_U|$$

The key addition layer can be omitted.

Automatic Search of Zero-Correlation Linear Hulls --Sketch of Searching ZCLHs

Sketch of searching ZCLHs

- Build a linear mask propagation system to catch all linear trails with bias
- A ZCLH is obtained if the corresponding linear mask propagation system has no solution.
- The techniques of predicting information and detecting contradictions are similar to those of finding impossible differentials.

——Algorithm for searching ZCLHs

Build the linear mask propagation system of a block cipher; for each pair of $(\Gamma P, \Gamma C)$ we choose do; 2. 3. *index*:=true; 4. while *index* do; 5. Predict information by soling the linear mask propagation system; if a contradiction is found then; 6. 7. *index*:=false; **return** true; 8. elseif cannot get any new information then; 9. *index*:=false; 10. end if; end while; 11. 12. end for;

Algorithm 1. Algorithm for searching ZCLHs

Suppose $\Gamma X = (\Gamma x_1, \Gamma x_2, ..., \Gamma x_n), \Gamma Y = (\Gamma y_1, \Gamma y_2, ..., \Gamma y_n)$ and $\Gamma Z = (\Gamma z_1, \Gamma z_2, ..., \Gamma z_n)$

Basi primit s		ΓX $\downarrow \qquad \qquad$		ΓX P \downarrow ΓY	$ \begin{array}{c} \Gamma X \\ \downarrow \\ S S \dots S \\ \downarrow \\ \Gamma Y \end{array} $
Equati	ions	$\Gamma x_i \oplus \Gamma y_i \oplus \Gamma z_i = 0$	$\Gamma x_i \oplus \Gamma y_i = 0,$ $\Gamma x_i \oplus \Gamma z_i = 0$	$\Gamma Y^t = (P^t)^{-1} \cdot \Gamma X^t$	$\overline{S}(\Delta x_i, \Delta y_i) = 0$
Not	e	dual property of the difference propagation and the linear mask propagation		$\Delta Y^t = P \cdot \Delta X^t$	linear distribution table (LDT)

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——Linear mask propagation system

Block ciphers	Linear mask propagation system	Note
$ \begin{array}{c c} \Gamma X_{i-1} & \Gamma X_i \\ \hline \Gamma Z_i & \Gamma Y_i \\ \hline \vdots \\ \Gamma X_i & \Gamma X_{i+1} \end{array} $	$\begin{cases} \overline{S}(\Gamma Z_{i,j}, \Gamma Y_{i,j}) = 0, \\ \Gamma Y_i^T \bigoplus P^t \cdot \Gamma X_i^T = 0, \\ \Gamma X_{i-1} \bigoplus \Gamma Z_i \bigoplus \Gamma X_{i+1} = 0 \end{cases}$ $(1 \le i \le r, 1 \le j \le n)$	N=(3r+2)n variables 2rn linear equations rn non-linear equations from Sbox layers
and the second state	Difference propagation system	
$\Delta X_{i} \qquad \Delta X_{i-1}$	$\begin{cases} \overline{S}(\Delta X_{i,j}, \Delta Y_{i,j}) = 0, \\ \Delta Z_i^T \oplus P \cdot \Delta Y_i^T = 0, \\ \Delta X_{i-1} \oplus \Delta Z_i \oplus \Delta X_{i+1} = 0 \end{cases}$ $(1 \le i \le r, 1 \le j \le n)$	N=(3r+2)n variables 2rn linear equations rn non-linear equations from Sbox layers

Outline

>Introduction

> Automatic Search of Impossible Differentials

Automatic Search of Zero-Correlation Linear Hulls

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Discussions and Conclusions

Discussions and Conclusions

——Discussions

Correctness and effectiveness

- IDs and ZCLHs found by our tool must be correct
- Finds the longest IDs and ZCLHs of many block ciphers
- Besides Sbox based designs, the idea can be extended to other designs, e.g., ARX block ciphers.
- Although our tool is more powerful than previous tools, it may also miss some longer IDs and ZCLHs
 - Only partially solve the difference/linear mask propagation system
 - Enumerating all possible pairs of $(\Delta P, \Delta C)$ and $(\Gamma p, \Gamma c)$ is beyond our computational ability
 - May miss some other conditions for detecting contradictions

Discussions and Conclusions

➢ Conclusions

- A new tool for searching IDs and ZCLHs is introduced.
- It helps in closing the gap between previous tools and ad hoc approaches
- For designers : evaluate the security of their block ciphers against impossible differential cryptanalysis and zerocorrelation linear cryptanalysis
- For attackers: find longer or new IDs and ZCLHs, which may improve known attacks.

Future work

Use more information of nonlinear equations or find more general filtering conditions to detect contradictions



Thank you for your attention!

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