

# Integral Attack Goes More than Impossible Differential Attack for LBlock

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This talk contains;

- Re-announcement of our SAC 2012 paper
  - Meet-in-the-Middle Technique for Integral Attacks against Feistel Ciphers (Yu Sasaki and Lei Wang)
- Recent updates on integral analysis for LBlock

# Status of SAC 2012 Paper

- Use MitM technique to reduce the complexity of integral attacks.
- Well-applied if the target is Feistel ciphers.

Target	Rounds	Data	Time	Ref.
LBlock	20	( Flawed )		[WZ11]
	20	$2^{63.6}$	$2^{39.6}$	<b>Ours</b>
HIGHT	22	$2^{62}$	$2^{118.71}$	[ZSL09]
	22	$2^{62}$	$2^{102.35}$	<b>Ours</b>
CLEFIA-128	12	$2^{115.7}$	$2^{116.7}$	[LWZ11]
	12	$2^{115.7}$	$2^{103.1}$	<b>Ours</b>

(#rounds are smaller than impossible diff. attacks.)

# Contents

- Integral Attack and Partial-Sum Technique
- MitM technique for Integral Attacks
- Applications to LBlock, HIGHT, and CLEFIA-128
- Concluding Remarks

# Introduction of Integral Attacks

- Introduced by Daemen et al. to evaluate the security of SQUARE cipher.
- Consisting of 2 parts;
  - Integral distinguisher
  - Key recovery phase

# Integral Distinguisher

- Prepare a set of plaintexts which contains all possible values (**A**) for some bytes and has a constant value (**C**) for the other bytes.

**(A, C, C, C, C, C, C, C)**



Several Rounds

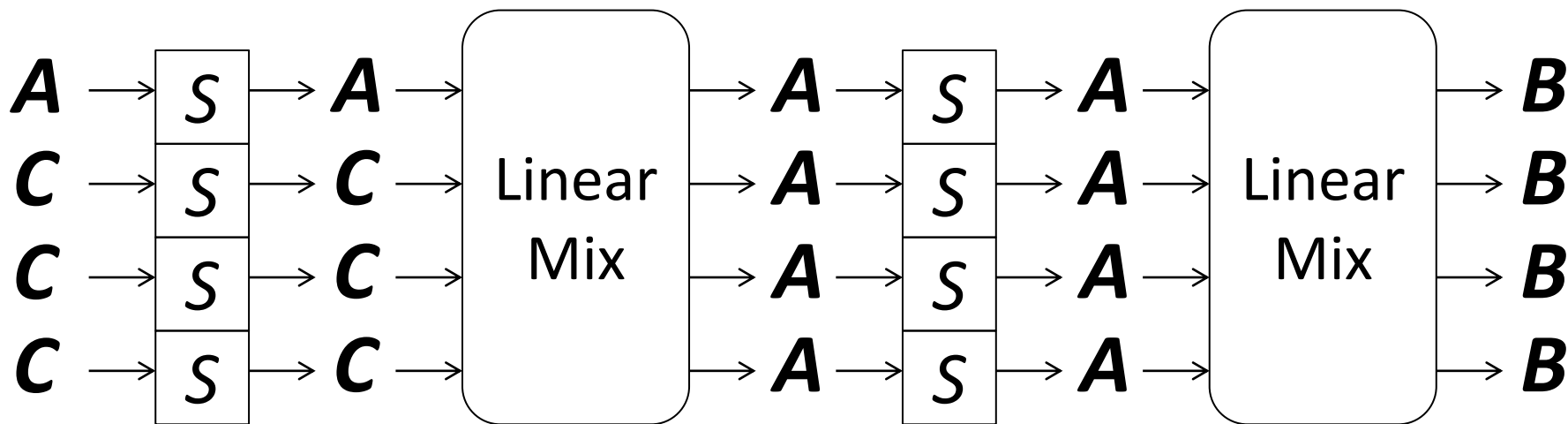


**(?, ?, ?, B, ?, ?, ?, ?)**

- XOR of all texts in the set becomes 0 (**B**).

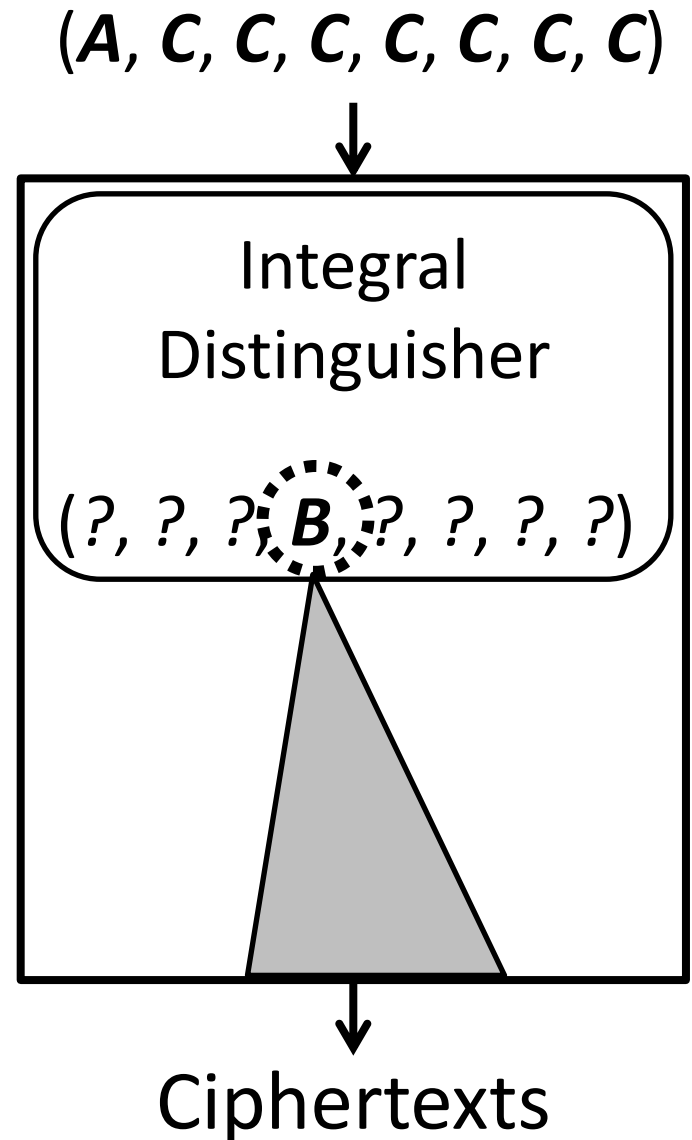
# Logics in Behind

- **A** and **C** are preserved through S-box.
- Mixing two **A** states lose its property, but still keeps **B** property.
- Such property can be traced for a few rounds.



# Key Recovery Phase

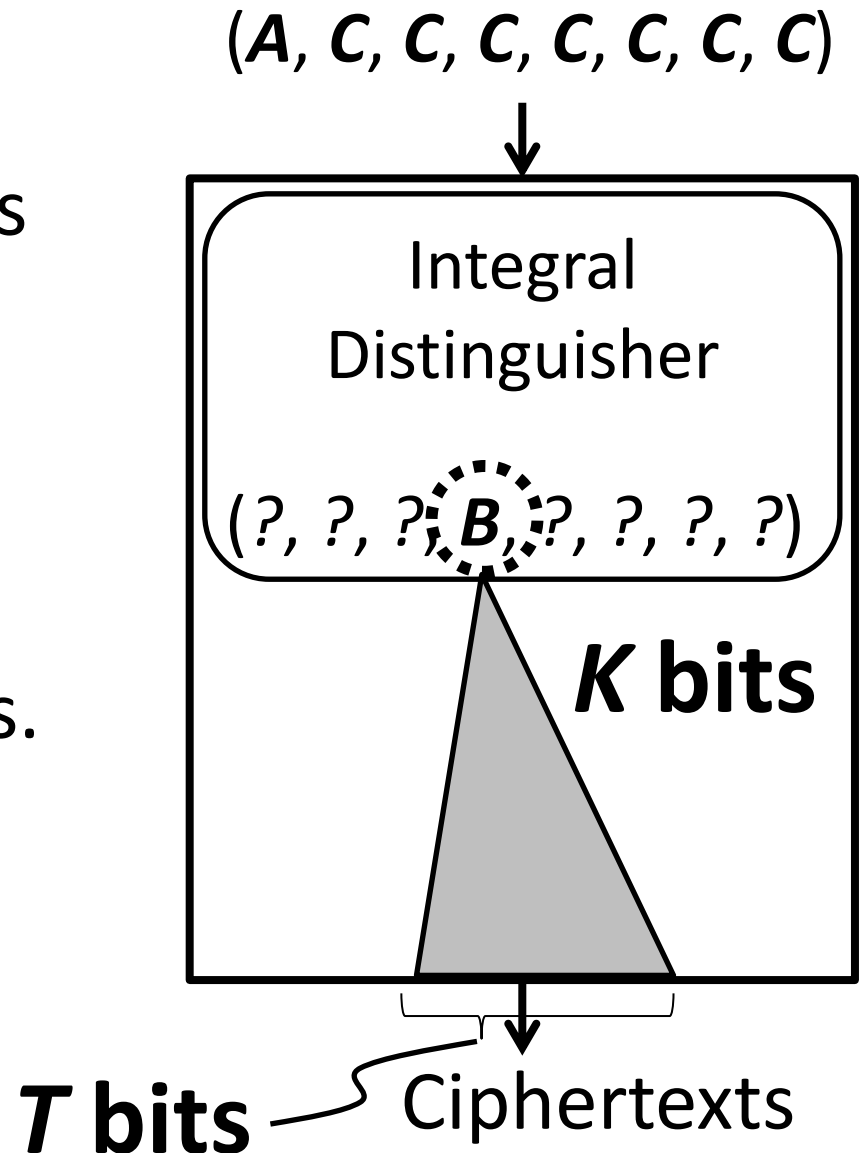
- Append several rounds to the distinguisher.
- Partially decrypt ciphertexts until the balanced state by partially guessing subkeys.
- If guess is correct, the sum of the results always becomes 0.



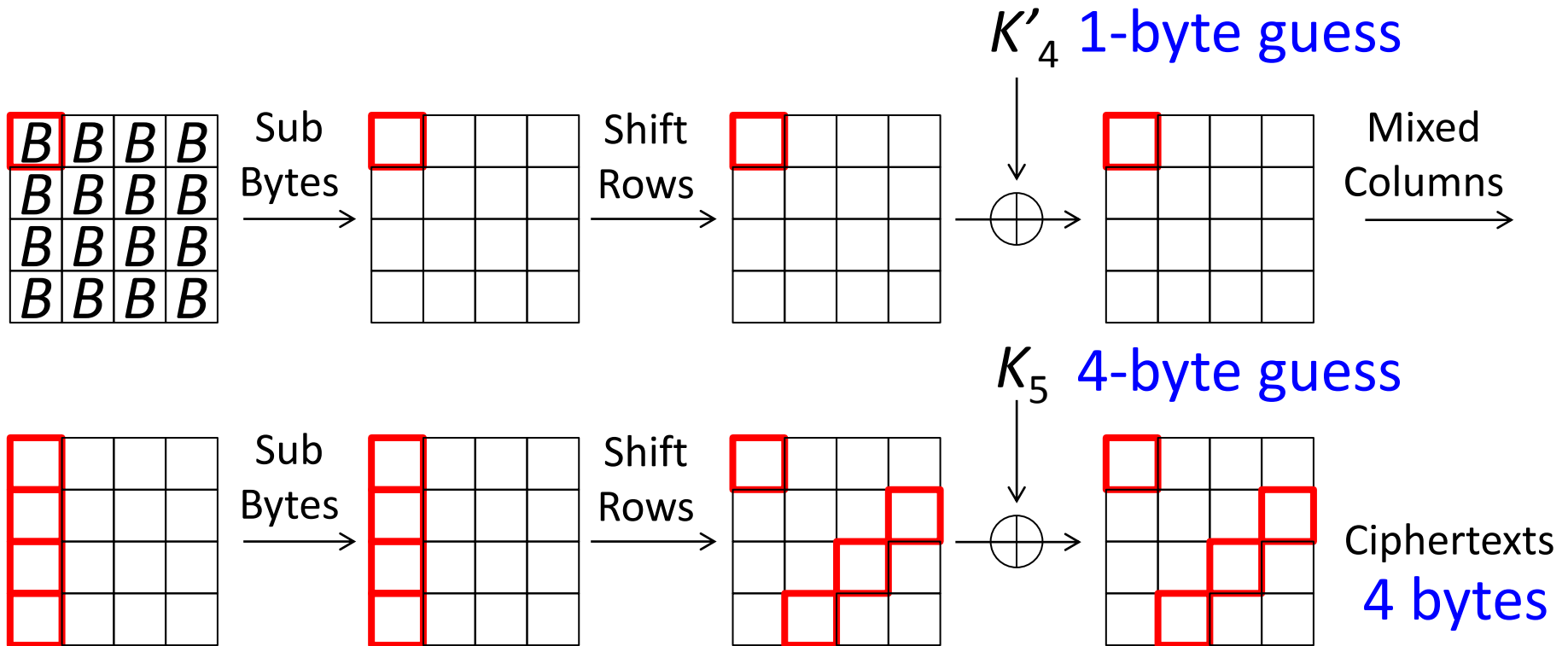
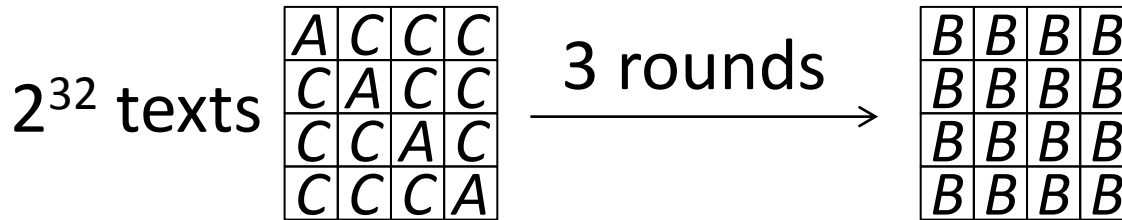


# Complexity of Integral Attack

- Suppose that the partial decryption involves  $T$  bits of ciphertexts and  $K$  bits of keys.
- It requires  $2^{T+K}$  partial decryption computations.
- Key space is reduced by  $|B|$  bits per set.



# Application to 5-round AES



$$\bigoplus_{n=1}^{2^{32}} \left[ S_4 \left( S_0(c_{0,n} \oplus k_0) \oplus S_1(c_{1,n} \oplus k_1) \oplus S_2(c_{2,n} \oplus k_2) \oplus S_3(c_{3,n} \oplus k_3) \oplus k_4 \right) \right]$$

# NTT Application to 5-round AES

$$\bigoplus_{n=1}^{2^{32}} \left[ S_4 \left( S_0(c_{0,n} \oplus k_0) \oplus S_1(c_{1,n} \oplus k_1) \oplus S_2(c_{2,n} \oplus k_2) \oplus S_3(c_{3,n} \oplus k_3) \oplus k_4 \right) \right]$$

Involves 4 ciphertext bytes and 5 key bytes.

- Straightforward:  $2^{32} * 2^{40} = 2^{72}$  computations.
- Partial-sum:  $2^{48}$  computations.

Guess each key byte one after another

# Partial-Sum Technique

$$\bigoplus_{n=1}^{2^{32}} \left[ S_4 \left( \underbrace{S_0(c_{0,n} \oplus k_0)}_x \oplus \underbrace{S_1(c_{1,n} \oplus k_1)}_y \oplus S_2(c_{2,n} \oplus k_2)}_z \oplus S_3(c_{3,n} \oplus k_3) \oplus k_4 \right) \right]$$

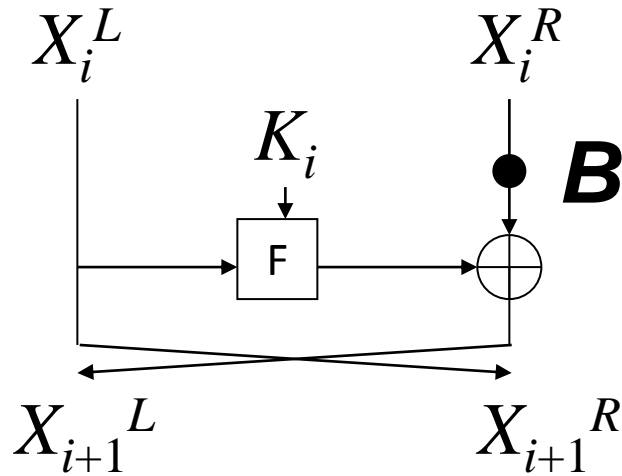
- Computation starts from  $2^{32}$  texts  $(c_0, c_1, c_2, c_3)$ .
  - Guess two key bytes  $k_0, k_1$ . Time:
  - For each guess, compute  $2^{32}$  tuples  $(x, c_1, c_2)$ .  $2^{32} * 2^{16} = 2^{48}$
  - Only pick  $(x, c_1, c_2)$  which appear odd times.
  - The size of the set is compressed into  $2^{24}$ .
    - Guess one key byte  $k_2$ . Time:
    - For each guess, compute  $2^{24}$  tuples  $(y, c_2)$ .  $2^{24} * 2^{16} * 2^8 = 2^{48}$
    - Only pick  $(y, c_2)$  which appear odd times.
    - The size of the set is compressed into  $2^{16}$ .

# Summary of Partial-Sum

- A technique to reduce the complexity of the key recovery phase for integral attacks.
- Whether or not it can be applied depends on the structure of the attack target.
- Our technique (MitM) can be combined with the partial-sum technique.

# Integral Analysis for Feistel Ciphers

- The property of  $\mathbf{B}$  is always broken from the right-hand side.

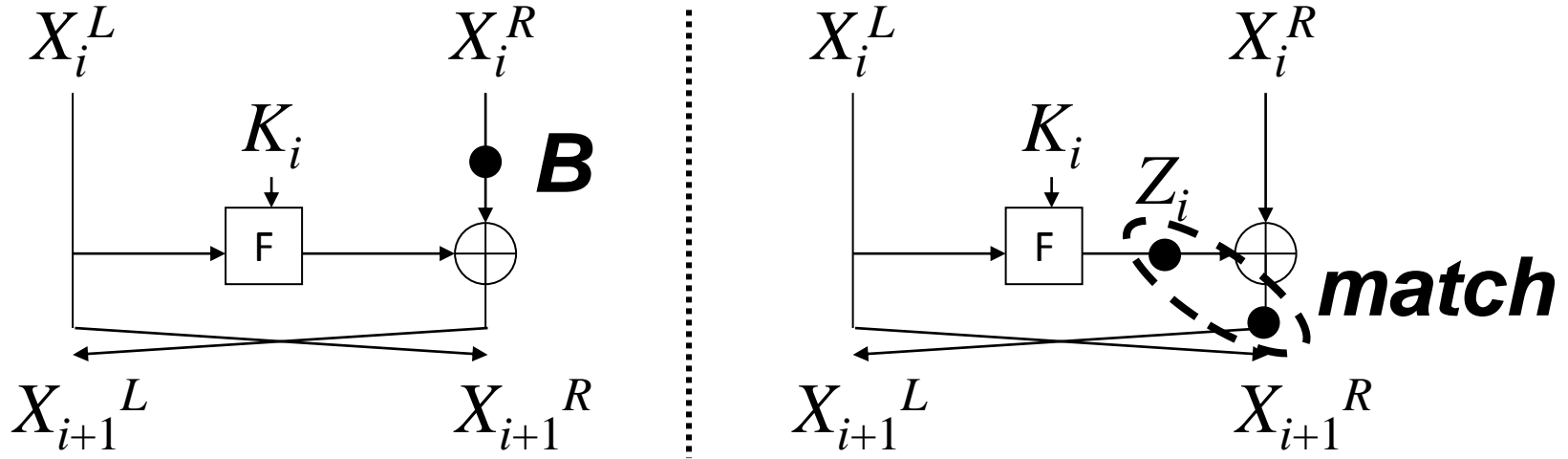


- Let  $\#K(X)$  and  $\#T(X)$  be the number of key bits and ciphertext bits to compute  $X$ .
- The complexity is  $2^{\#K(X)+\#T(X)}$ .

# MitM technique for Integral Attacks

The same approach was used in the  
designers' evaluation of TWINE.

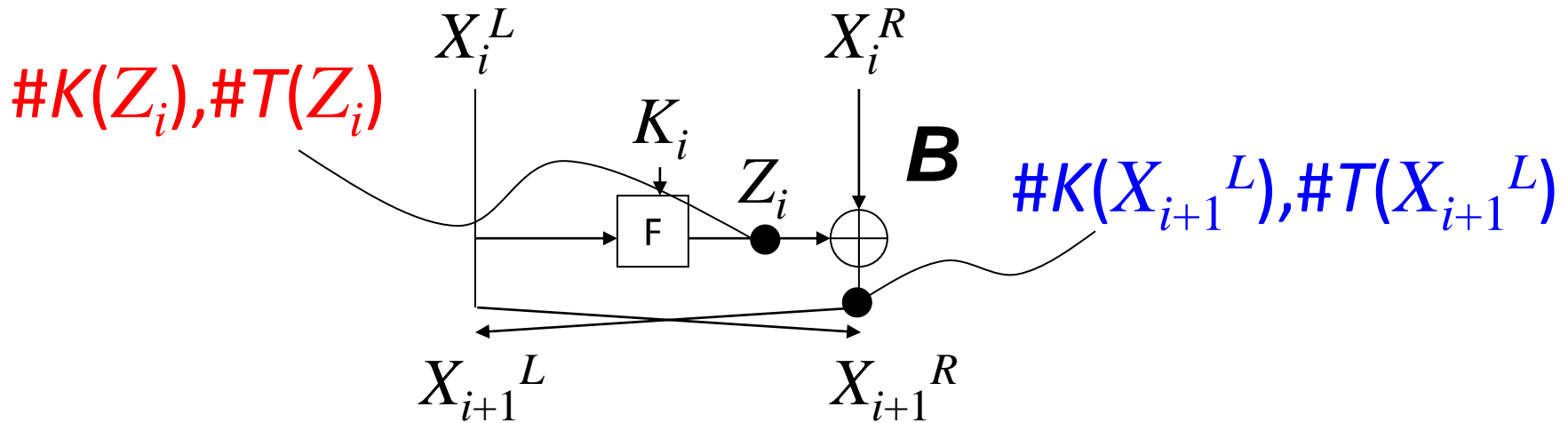
# Introduction of MitM Approach



- $\bigoplus X_i^R = 0 \Rightarrow \bigoplus (Z_i \oplus X_{i+1}^L) = 0$   
 $\Rightarrow \bigoplus Z_i = \bigoplus X_{i+1}^L$
- Two terms can be computed independently.  
 Right key candidates are identified by checking the match of two lists.



# Complexity of the MitM Approach

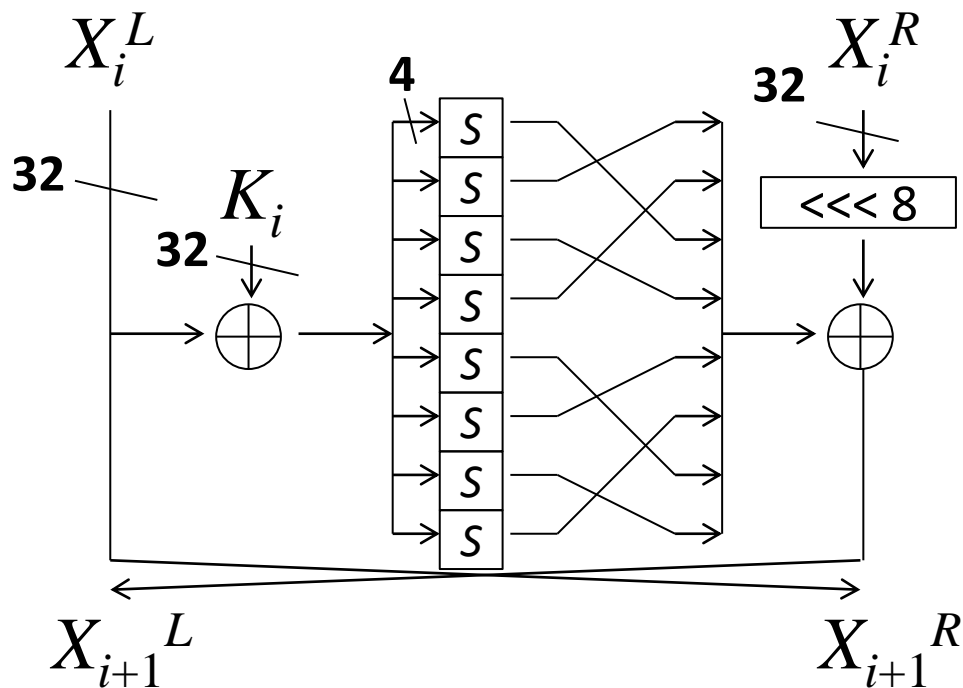


- Red part is always more expensive than blue part.
- The complexity is  $2^{\#K(Z_i) + \#T(Z_i)}$ .
- If  $\#K(Z_i)$  and  $\#K(X_{i+1}^L)$  share some bits in common, memory complexity can be saved as standard meet-in-the-middle attacks.

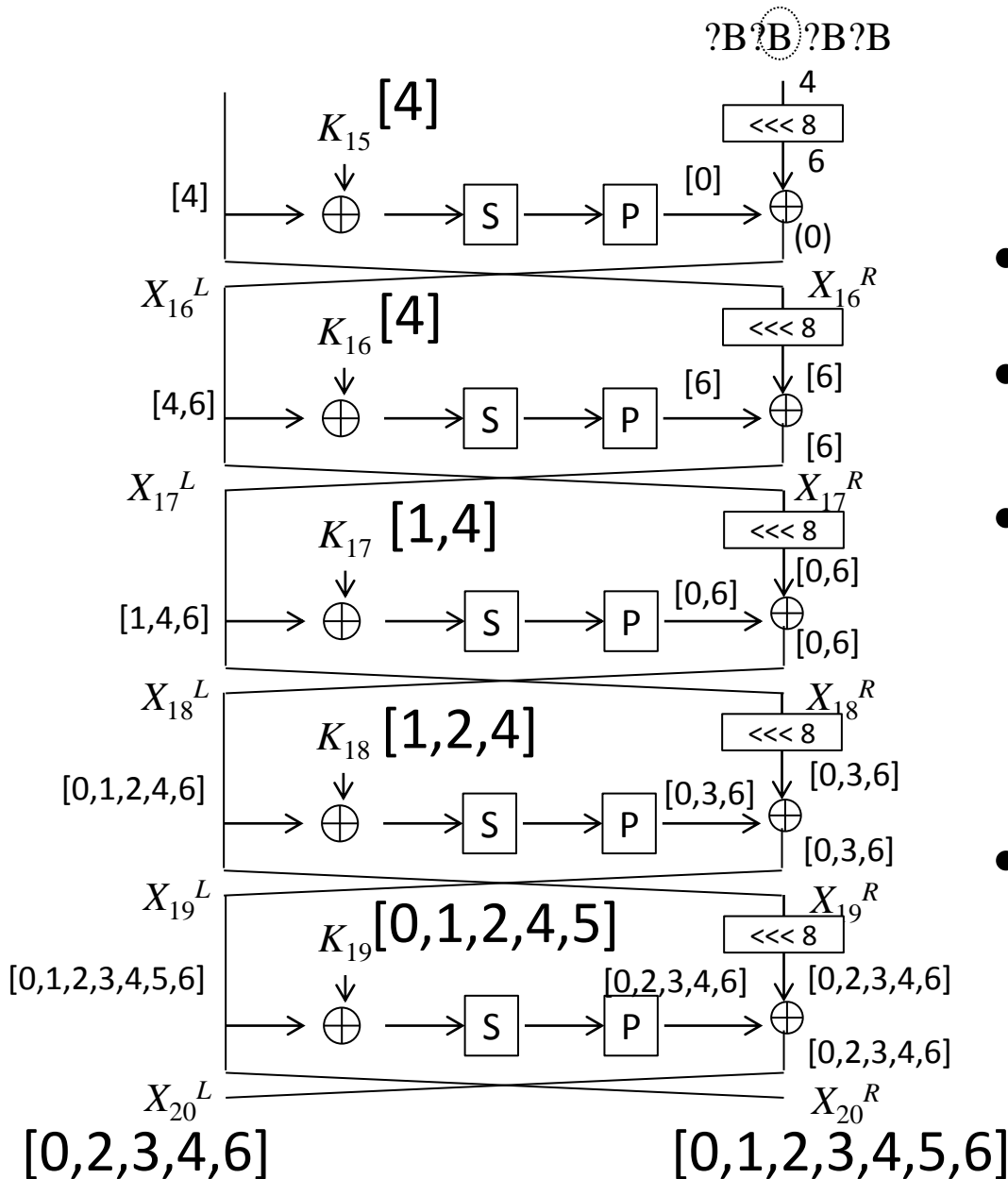
# Applications

# LBlock

- Proposed by Wu and Zhang at ACNS 2011.
- 64-bit block, 80-bit key.
- Modified Feistel structure with 32 rounds.
- 15-round distinguisher is known.

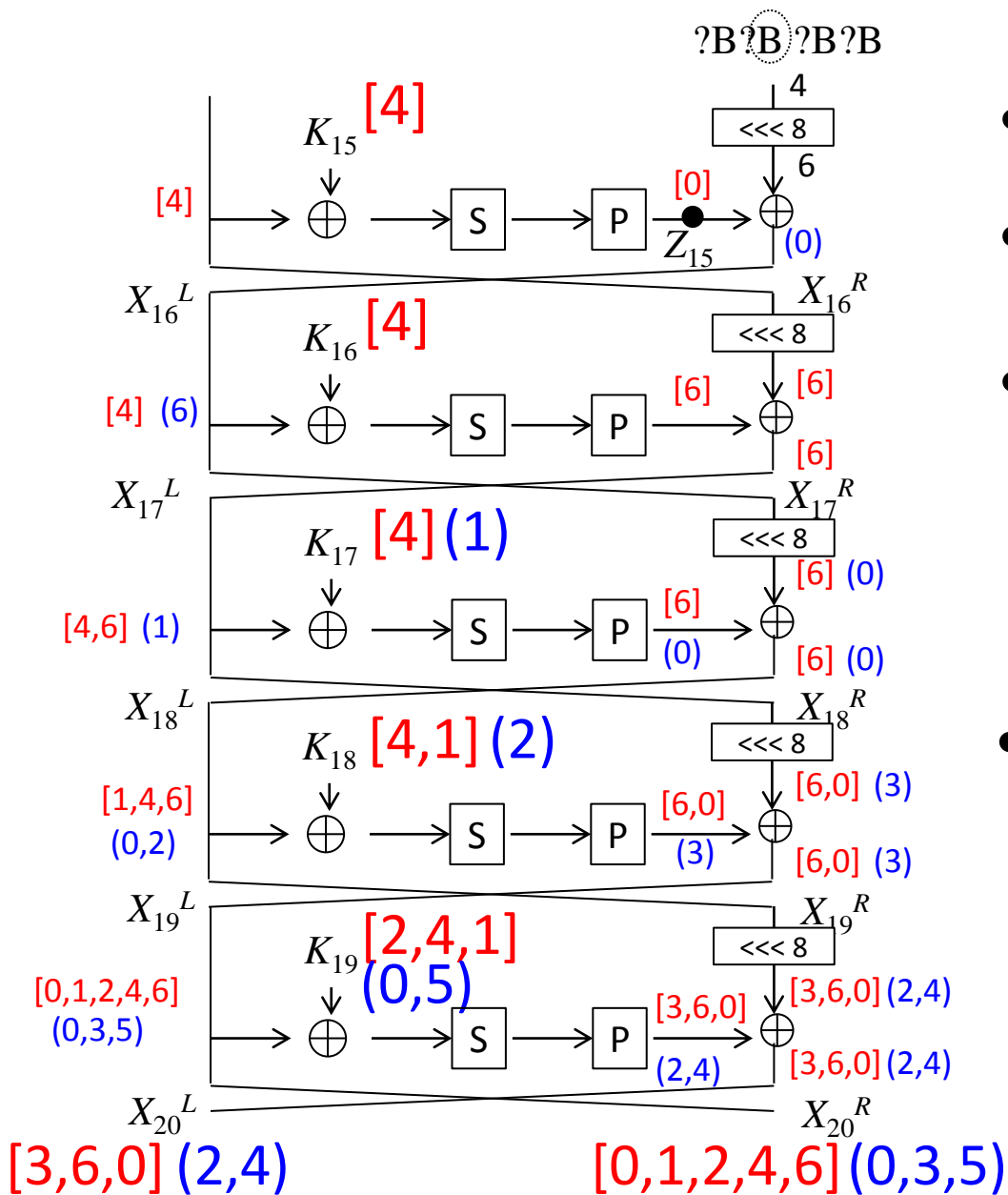


# Flaw of Previous 20-Round Attack



- $\#K(X_{15}[4])=48$
- $\#T(X_{15}[4])=48$
- The complexity is  $2^{48+48} = 2^{96}$ , worse than the brute force.
- [WZ11] did not count  $\#T(X_{15}[4])$ .

# Our 20-Round Attack



- $\#K(Z_{15})=32$

- $\#T(Z_{15})=32$

- The complexity is  $2^{32+32} = 2^{64}$ .

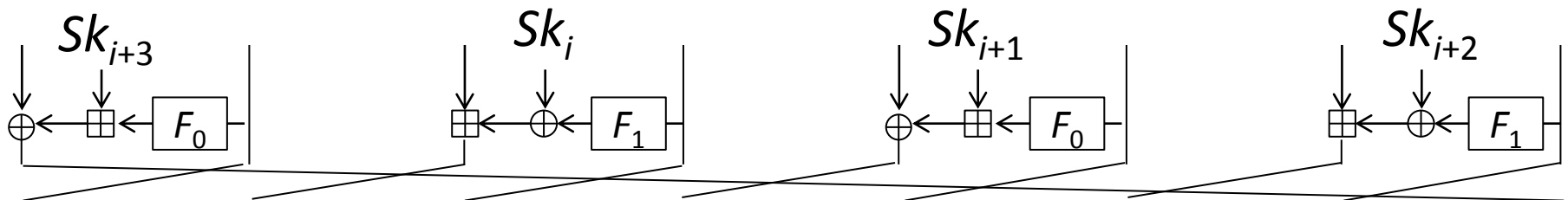
Valid Attack !!

- Further optimization by the *partial-sum*:

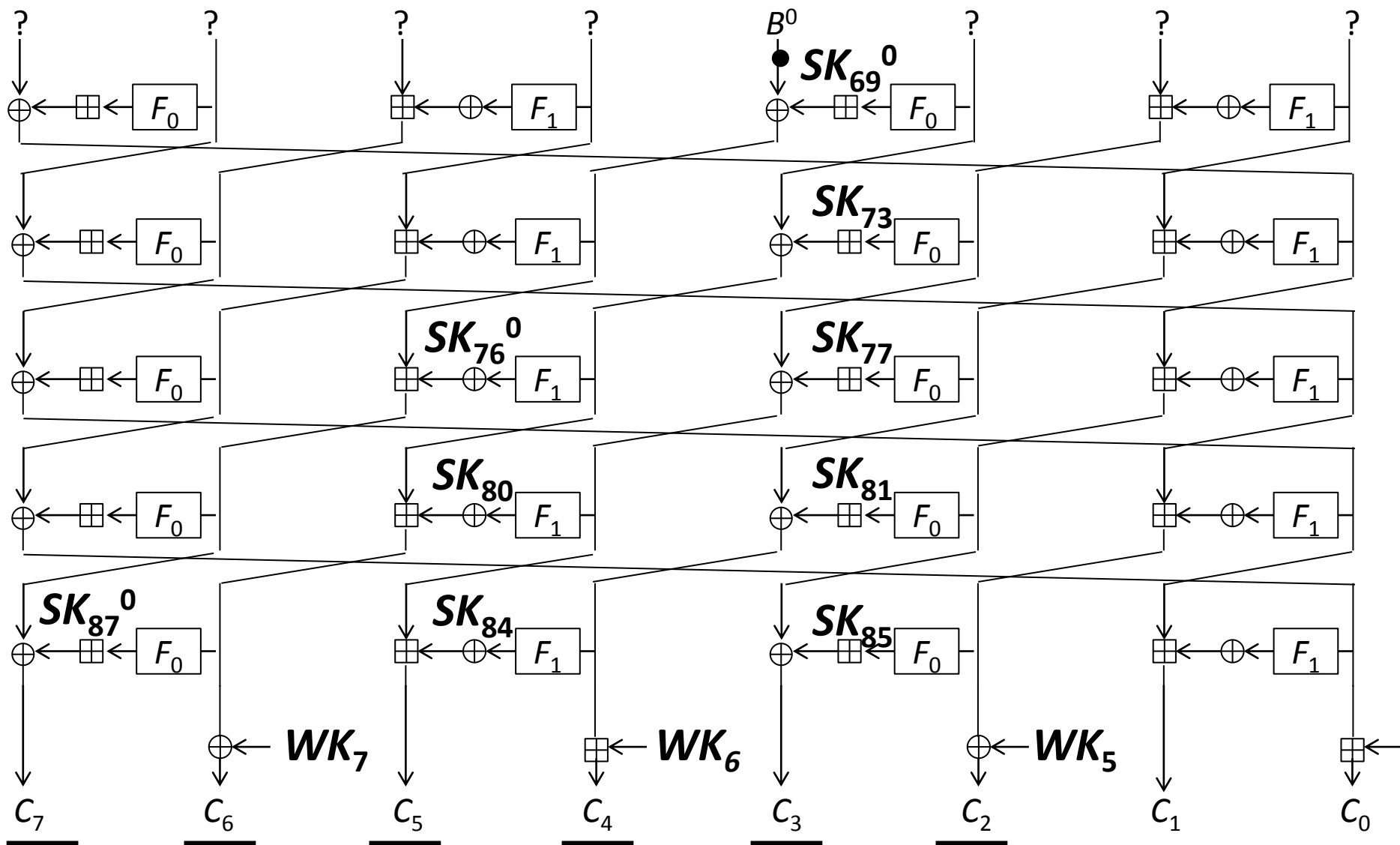
Complexity is  $2^{36}$ .

# HIGHT

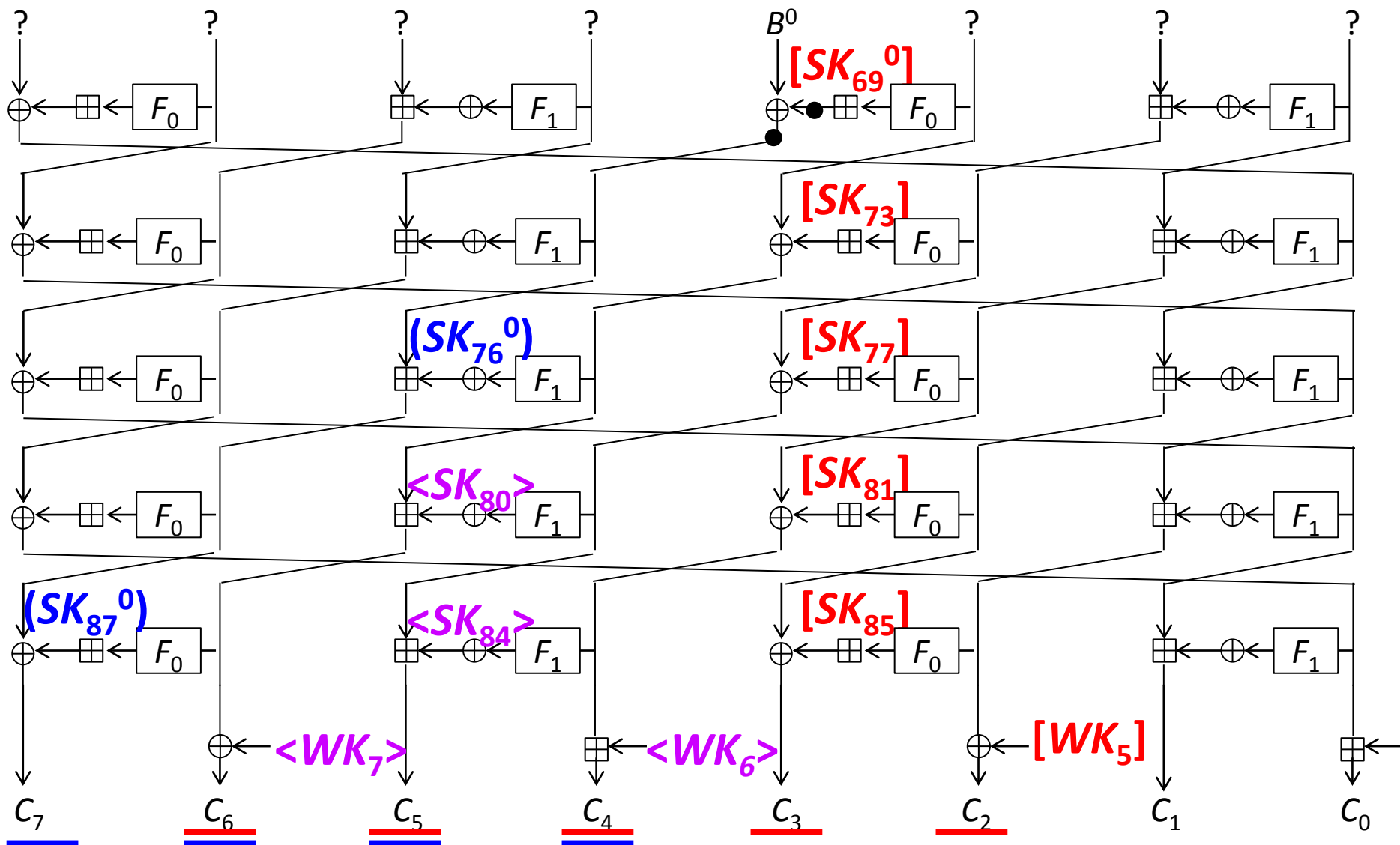
- Proposed by Hong et al. at CHES 2006.
- 64-bit block, 128-bit key.
- Generalized Feistel network with 8 branches, in total 32 rounds.
- 17-round distinguisher is known.



# 22-Round Attack on HIGHT



# 22-Round Attack on HIGHT

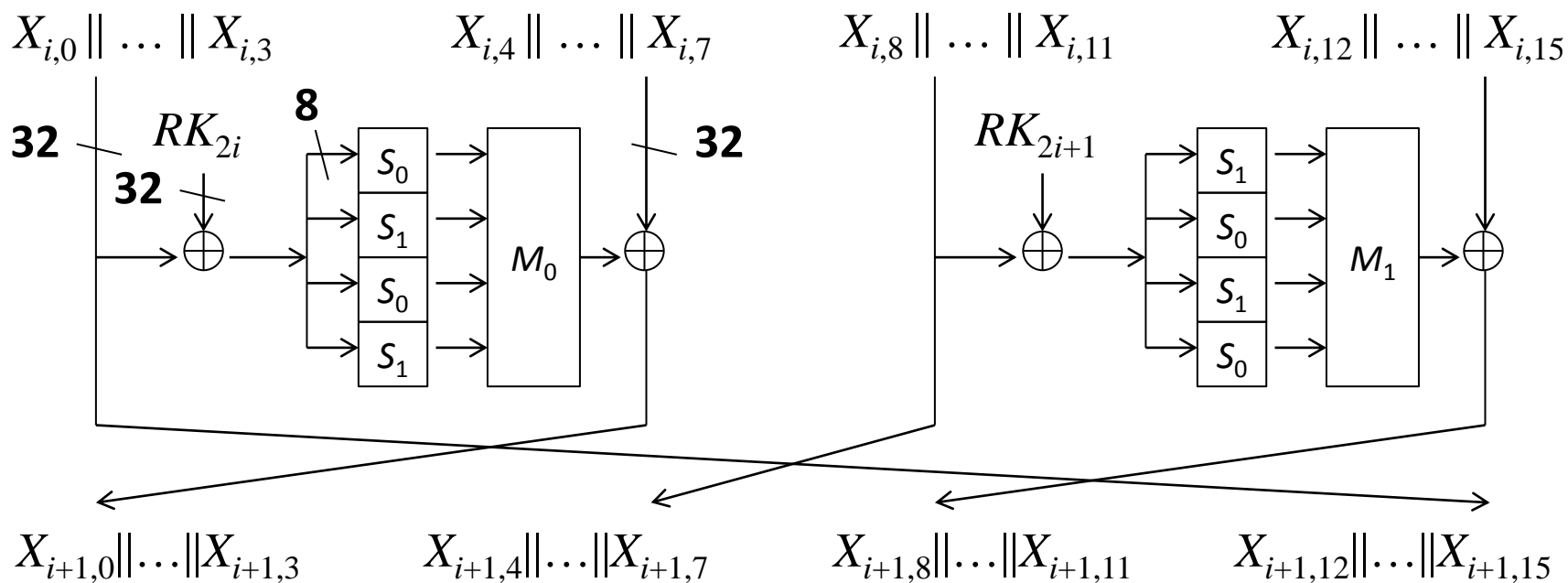


Our improvement contains other small observations.



# CLEFIA-128

- Proposed by Shirai et al. at FSE 2007.
- 128-bit block, 128-bit key.
- Generalized Feistel network with 4 branches, in total 18 rounds.
- 9-round distinguisher is known.



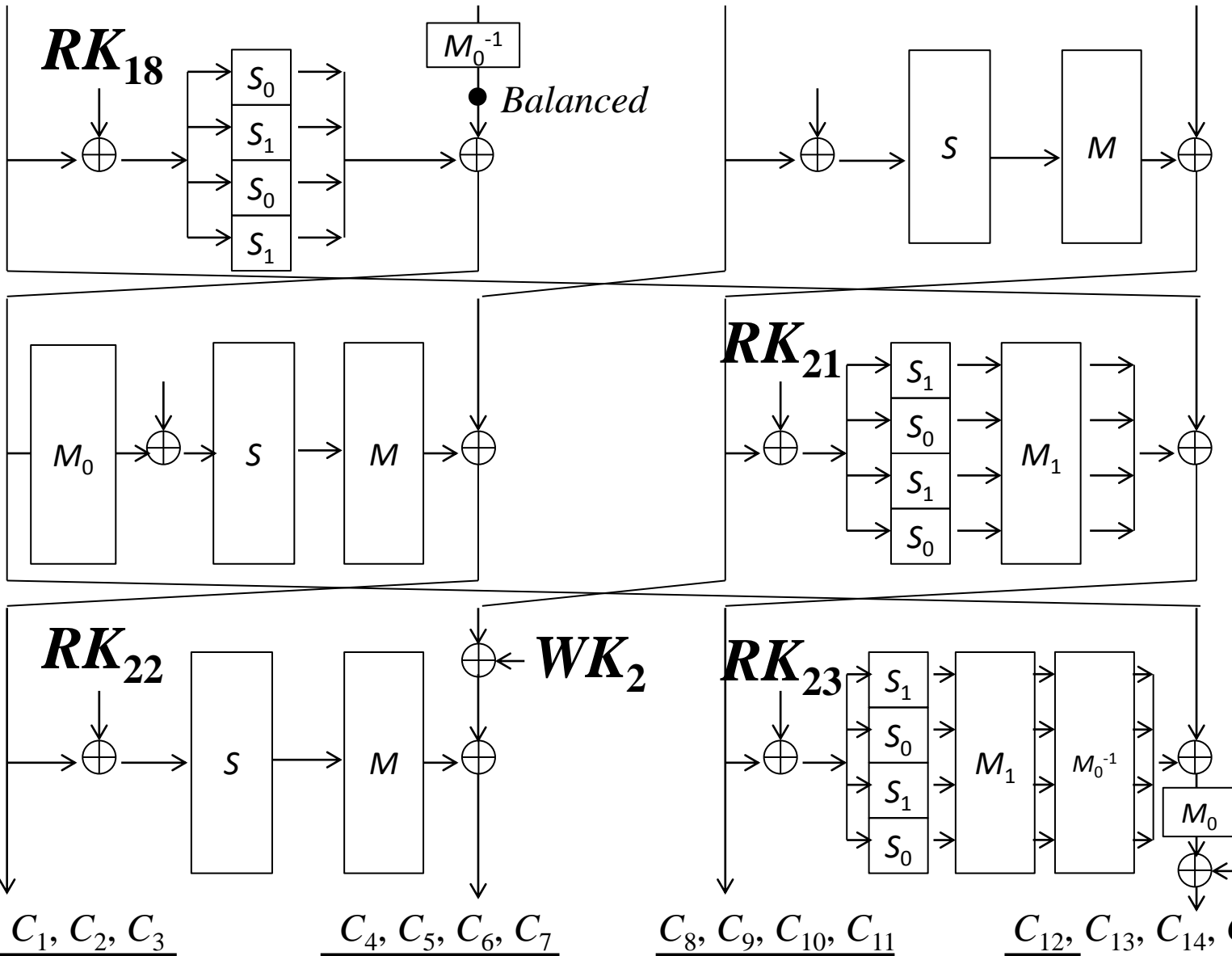
# 12-Round Attack on CLEFIA-128

?, ?, ?, ?

$B, B, B, B$

?, ?, ?, ?

?, ?, ?, ?



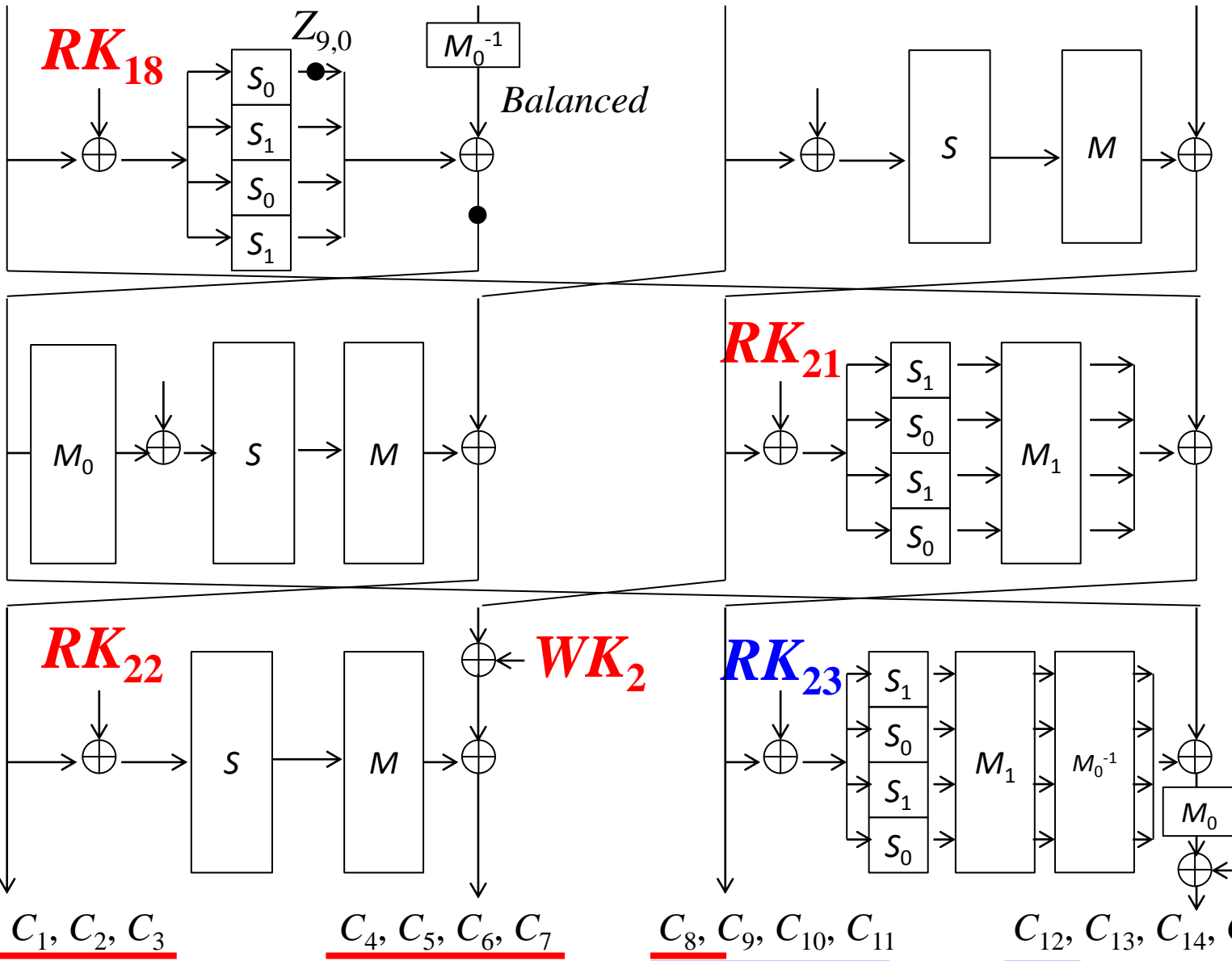
# 12-Round Attack on CLEFIA-128

?, ?, ?, ?

$B, B, B, B$

?, ?, ?, ?

?, ?, ?, ?



# Partial-Sum for Feistel Ciphers

- We also improve the partial-sum on CLEFIA, which is applied to Feistel ciphers in generic.

$$\begin{aligned} & \bigoplus \left[ S_0 \left( \underbrace{S_1(C_8 \oplus RK_{21,0}) \oplus 08 \cdot S_0(C_9 \oplus RK_{21,1})}_{x} \oplus \right. \right. \\ & \quad \left. \left. 02 \cdot S_1(C_{10} \oplus RK_{21,2}) \oplus 0a \cdot S_0(C_{11} \oplus RK_{21,3}) \oplus C_{12} \oplus RK'_{18,0} \right) \right] \\ & = \bigoplus C', \end{aligned} \tag{8}$$

- **Previous:** Guess 2 key bytes, and then compress.

- 

**Previous:**  $2^{40+16}=2^{56}$

# NTT Partial-Sum for AES [reuse]

$$\bigoplus_{n=1}^{2^{32}} \left[ S_4 \left( \underbrace{S_0(c_{0,n} \oplus k_0)}_x \oplus \underbrace{S_1(c_{1,n} \oplus k_1)}_y \oplus \underbrace{S_2(c_{2,n} \oplus k_2)}_z \oplus S_3(c_{3,n} \oplus k_3) \oplus k_4 \right) \right]$$

- Computation starts from  $2^{32}$  texts  $(c_0, c_1, c_2, c_3)$ .
  - Guess two key bytes  $k_0, k_1$ . **Time:**
  - For each guess, compute  $2^{32}$  tuples  $(x, c_1, c_2)$ .  **$2^{32} * 2^{16} = 2^{48}$**
  - Only pick  $(x, c_1, c_2)$  which appear odd times.
  - The size of the set is compressed into  $2^{24}$ .
    - Guess one key byte  $k_2$ . **Time:**
    - For each guess, compute  $2^{24}$  tuples  $(y, c_2)$ .  **$2^{24} * 2^{16} * 2^8 = 2^{48}$**
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# Partial-Sum for Feistel Ciphers

- We also improve the partial-sum on CLEFIA, which is applied to Feistel ciphers in generic.

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- Previous:** Guess 2 key bytes, and then compress.
- Ours:** Feistel ciphers usually includes the term which only consists of ciphertext.

Guess only 1 byte, and then compress.

**Previous:**  $2^{40+16}=2^{56}$

**Ours:**  $2^{40+8}=2^{48}$

# Summary of Our SAC Paper

- Use MitM approach for integral attacks.
- Well-applied to Feistel ciphers.
- Applied it to LBlock, HIGHT, and CLEFIA-128 together with other improvements.

# Integral Attack Goes More than Impossible Differential Attack for LBlock



# *Life is so HARD!*

Target	Rounds	Data	Time	Ref.
Imp. Diff.	21	$2^{62.5}$	$2^{73.7}$	[LG++11]
	20	$2^{63.6}$	$2^{39.6}$	[SW12]
Integral	21	$2^{61.6}$	$2^{54.2}$	<b>Ours</b>
	22	$2^{61}$	$2^{70.0}$	<b>Ours</b>
Biclique	32	$2^{52.7}$	$2^{78.4}$	[WW++12]

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Target	Rounds	Data	Time	Ref.
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	21	$2^{61.6}$	$2^{54.2}$	<b>Ours</b>
	22	$2^{61}$	$2^{70.0}$	<b>Ours</b>
	<b>22</b>	<b><math>2^{61.6}</math></b>	<b><math>2^{71.2}</math></b>	<b>[L12]*</b>
Biclique	32	$2^{52.7}$	$2^{78.4}$	[WW++12]

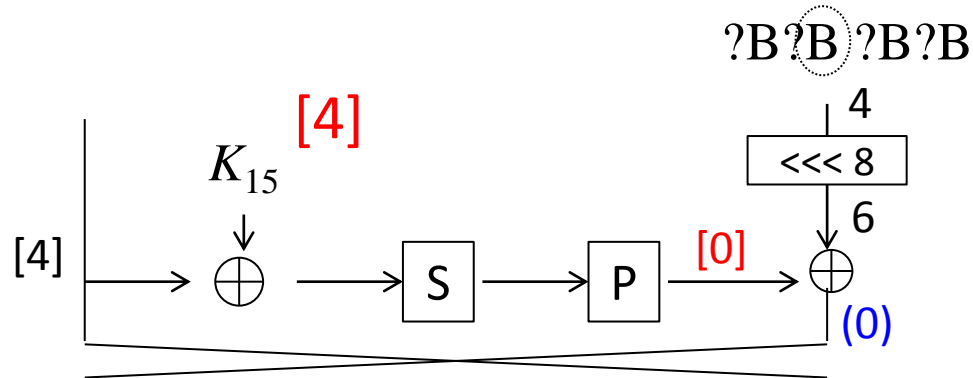
\*: Yanjun Li. Integral Cryptanalysis on Block Ciphers (in Chinese): [D]. Beijing: Institute of Software, Chinese Academy of Sciences, 2012.

*We knew that*

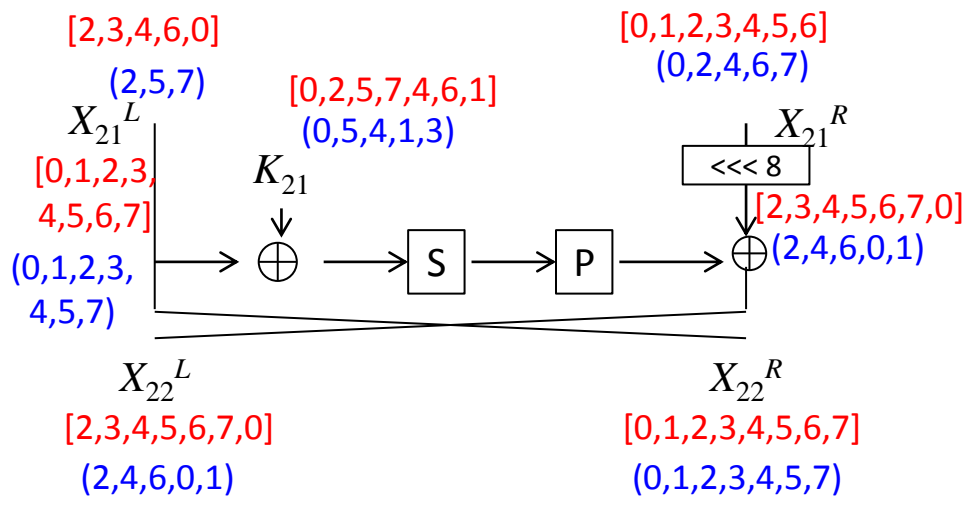
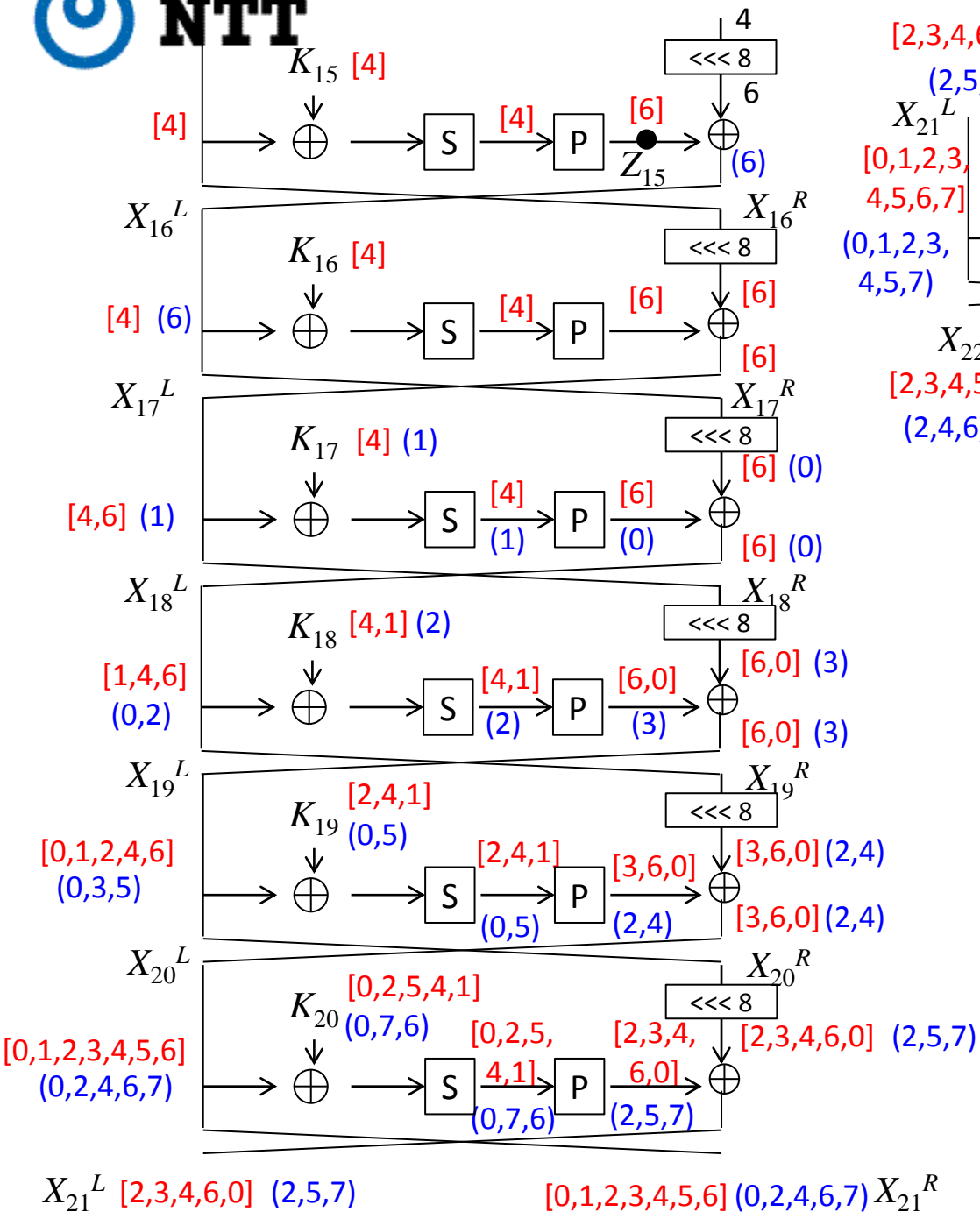
# Integral Attack Goes More than Impossible Differential Attack for LBlock

The contents in this talk are completely independent of the results in [L12].

# Comprehensive Analysis



- Try all possible balanced byte positions.
- Meet-in-the-Middle technique
- Partial-sum technique
- Subkey relations
- Combining exhaustive search

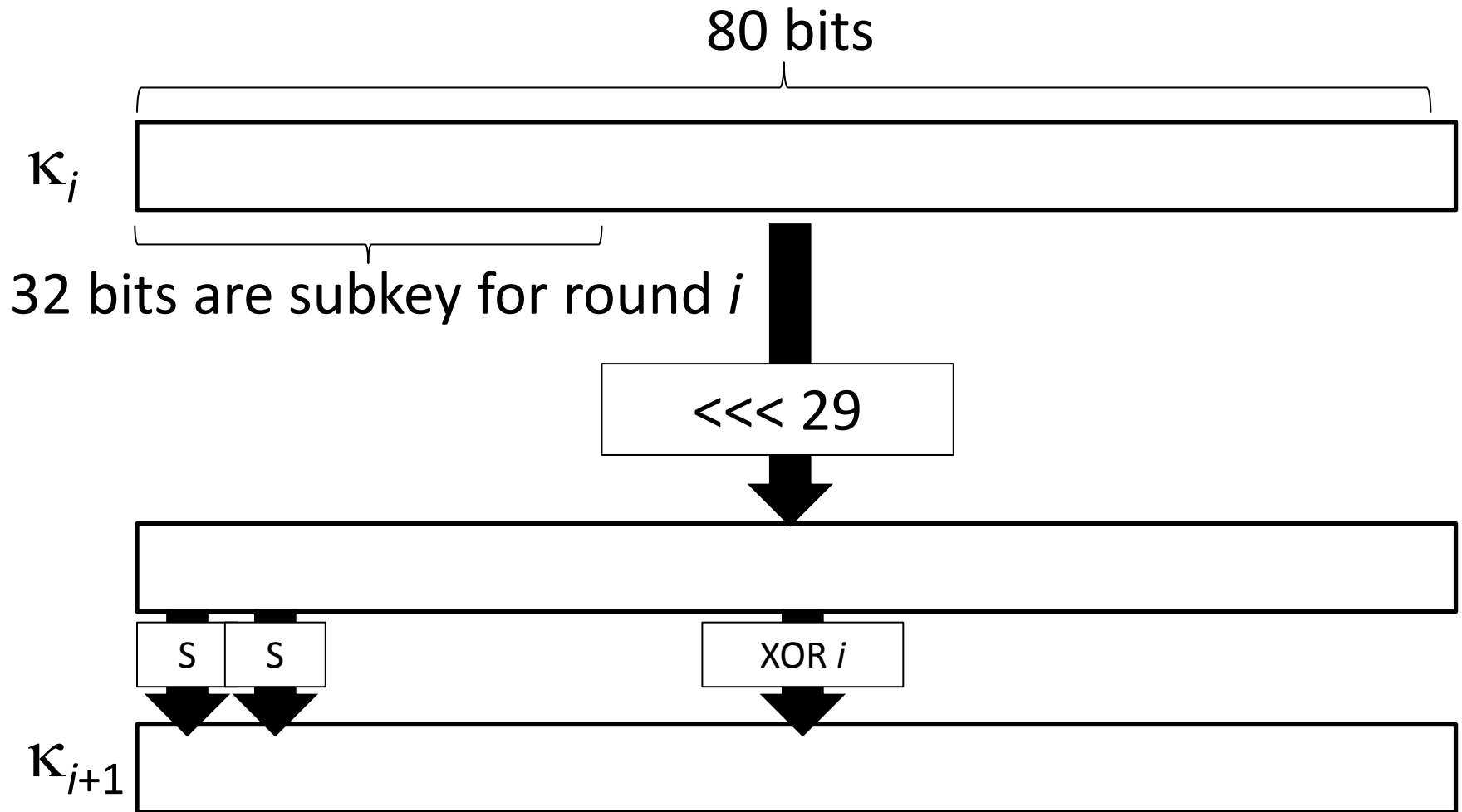


# Overview

- #Guessed key bytes becomes 32 bytes (20 bytes for the red part, 12 bytes for the blue part)
- #related ciphertext bytes are also many (15 bytes for the red part and 12 bytes for the blue part).
- Using the partial-sum is necessary, but still not enough to be a valid attack.
- Relations between subkeys must be considered.

# Considering the key schedule

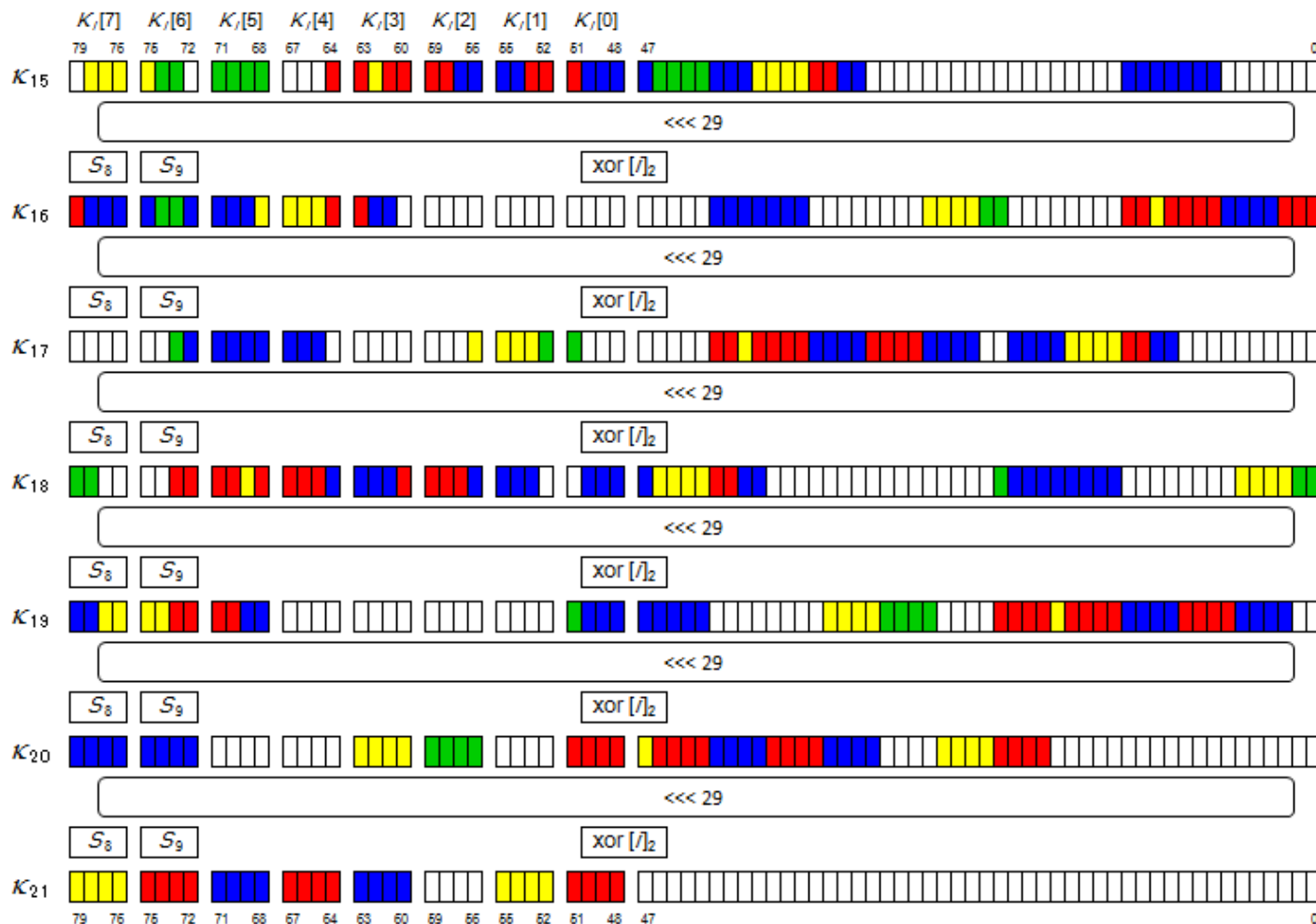
- Master key is loaded into the 80-bit key state  $\kappa_0$ .



# Key Schedule Analysis

#overlapped bits depends on guessed key-byte positions.

The analysis must be iterated for all balanced-byte positions.





# Identifying best balanced-byte position

Exhaustively checked which balanced-byte position is the best to mount an integral attack.

	<b>0th</b>	<b>2nd</b>	<b>4th</b>	<b>6th</b>
<b>21R red</b>	<b>50</b>	<b>44</b>	<b>47</b>	<b>42</b>
<b>21R both</b>	<b>63</b>	<b>61</b>	<b>63</b>	<b>57</b>
<b>22R red</b>	<b>62</b>	<b>55</b>	<b>63</b>	<b>65</b>
<b>22R both</b>	<b>75</b>	<b>69</b>	<b>75</b>	<b>77</b>

# Summary

- We did comprehensive analysis on LBlock. Optimize the attack with all exiting techniques.
- The number of attacked rounds is extended.
- First example that the integral analysis could beat the impossible differential analysis.

Target	Rounds	Data	Time	Ref.
Imp. Diff.	21	$2^{62.5}$	$2^{73.7}$	[LG++11]
Integral	22	$2^{61}$	$2^{70.0}$	<b>Ours</b>
	22	$2^{61.6}$	$2^{71.2}$	[L12]*

***Thanks for your attention !!***