Iterated Space-Time Code Constructions from Quaternion Algebras

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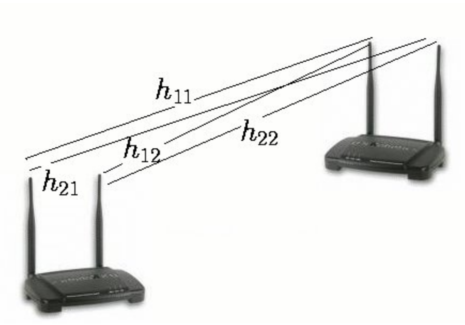
Talk Outline

MIMO Channel

Multiple Input Multiple Output



• Space-time codes: used in multiple-antennae systems for higher data rate and reliability over fading channels

System Model: MIDO codes

Multiple Input Double Output

- Application: Broadcasting to a portable device
- Consider: 4 Transmit, 2 Receive antannae R_1, R_2, perfect *CSIR*

At each time interval j,

- R_1 receives a superposition of signals $(x_{1j}, x_{2j}, x_{3j}, x_{4j})$ plus noise

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- To transmit 8 complex symbols, need coherence time of at least 4.

Putting this into matrix form we obtain:

$$Y_{2\times 4} = H_{2\times 4}X_{4\times 4} + N_{2\times 4}$$

H = channel matrix
X = space-time codeword
N = noise matrix
H, N are i.i.d. complex Gaussian

Algebras: a tool for STC

- *Full Diversity*: Want a collection of matrices so that

$$\min\{det(X - Y) : X, Y \in \mathscr{C}\} \neq 0$$

  this bounds pairwise probability of error (Tarokh et al)
- Algebras give rise to space-time codes
  - linearity
  - full diversity (in case algebra is division)
  - nice criteria for diversity

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- Algebras give rise to space-time codes
  - linearity
  - full diversity (in case algebra is division)
  - nice criteria for diversity
- Many examples of codes with good performance already known.
- Problem: Maximum Likelihood (*ML*) decoding complexity too high
- Need algebraic codes which are fast decodable

ML Decoding

- A 4×4 MIDO code can transmit up to 8 complex information symbols, so 16 real (say PAM) information symbols.
- Space-time code $\mathscr{C}$ is a vector space generated by matrices B_i.
- In our case $B_i \in Mat_{4\times 4}$ and $X \in \mathscr{C}$ has the form

$$X = \sum_{i=1}^{16} g_i B_i,$$

where g_i are the information symbols from a real constellation.

ML Decoding

Y = received matrix. Minimize the distance

$$\{d(X) = ||Y - HX||_F^2\}_{X \in \mathscr{C}} \quad (1)$$

QR Decomposition

- Each 4x4 matrix $B_i \mapsto HB_i \mapsto \mathbf{b}_i \in \mathbb{R}^{16} \cong \mathbb{C}^8$
- A 16-dimensional code gives rise the the matrix

$$B = (\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{16}) \in M_{16\times 16}(\mathbb{R}), \qquad (2)$$

- **QR decomposition:** $B = QR$, with $Q^\dagger Q = I$,

ML decoding

reduces to minimizing

$$d(X) = ||\mathbf{y} - QR\mathbf{g}||_E^2 = ||Q^\dagger \mathbf{y} - R\mathbf{g}||_E^2 \qquad (3)$$

- Complexity of exhaustive search is $O(|S|^{16})$, where $S =$ the constellation.
- *Fast decodable codes*: those which offer improvement on decoding complexity.

Fast Decodability

Complexity can be reduced if R is guaranteed to have a nice zero-structure.

Definition (Fast-decodable Codes)

A space-time code is said to be *fast-decodable* if its R matrix has the following form:

$$R = \begin{bmatrix} \Delta & B_1 \\ 0 & R_2 \end{bmatrix},$$

where Δ is a diagonal matrix and R_2 is upper-triangular.

Definition (g-group Decodable Codes)

A space-time code of dimension K is called *g-group decodable* if the matrix R has the form $R = diag(R_1, \ldots, R_g)$, where each R_i is a square upper triangular matrix.

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Definition

Given an ordering on $B_1, \ldots, B_K$, let M be a matrix capturing information about orthogonality relations of the basis elements of B_i:

$$M_{k,l} = ||B_k B_l^* + B_l B_k^*||_F. \tag{4}$$

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Nice zero structure of $M \implies$ nice zero structure of R. (Rajan et al.)

- $M = \begin{bmatrix} \Delta & B_1 \\ B_2 & B_3 \end{bmatrix}$, where Δ is diagonal $\implies R = \begin{bmatrix} \Delta & B_1 \\ 0 & R_1 \end{bmatrix}$.
- $M = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \implies R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}$.

Summarize Our Objective

- Codes arising from algebras.
- *Full Diversity*:

$$\min\{det(X-Y) : X, Y \in \mathscr{C}\} \neq 0$$

$$\Downarrow \text{ by linearity}$$

$$\min\{det(X) : X \in \mathscr{C}\} \neq 0$$

- *Fast Decodability*: Matrix R must have a nice zero-block structure
  Sufficient to find an ordering on basis elements of the code, so that M has a nice zero-structure.

Contribution: Iterated Codes

We propose an iterated construction of space-time codes

- Full-rate 4×4 MIDO codes
- Fast-decodable: complexity reduction from $O(|S|^{16})$ to $O(|S|^{13})$,$O(|S|^{10})$,$O(|S|^{8})$.
- Criteria for full diversity

Quaternion Algebras and 2×2 codes

Recall Hamiltonian Quaternions.

- $\mathbb{H}$=vector space of dimension 4 over $\mathbb{R}$, with basis

$$\{1, i, j, k\}.$$

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- Rules: $i^2 = -1$, $j^2 = -1$, $k = ij = -ji$.

An element $x \in \mathbb{H}$ can be written as

$$x = c + jd, \text{ where } c, d \in \mathbb{C}.$$

The resulting code $\mathscr{C}$ corresponds to the celebrated Alamouti code [1]

$$\mathscr{C} = \{ \begin{bmatrix} c & -d^* \\ d & c^* \end{bmatrix} \mid c, d \in \mathbb{Z}[i].\}$$

Generalized Quaternion Algebras and 2×2 Codes

Similarly

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$$\text{So } \mathbb{H} = (-1, -1)_{\mathbb{R}}$$

$$c + jd \mapsto \begin{bmatrix} c & \gamma\sigma(d) \\ d & \sigma(c) \end{bmatrix}.$$

- Code $\mathscr{C} = \{\lambda(x) : x \in \Lambda\}$, where Λ is an order of Q.

Codes from Quaternion Algebras

- Quaternion algebras are a special case of a cyclic algebra.
- Quaternion algebra $Q = (a, \gamma)_F$ is of degree 2 over its maximal subfield $K = F(\sqrt{a})$
- $Q \cong K \oplus jK$ with $j^2 = \gamma$. It is a *right* vector space over K
- Left regular representation gives matrices

$$\lambda : Q \to M_{2\times 2}(K)$$

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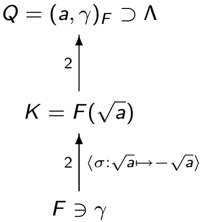

Codes from Algebras

In general

- Cyclic algebras are constructed from a number field extension K/F,

$$\mathscr{A} = (K/F, \sigma, \gamma)$$

- If $\mathscr{A}$ division, then resulting matrices are full rank, i.e., the code has full diversity
- Easy criterion for full diversity in terms of γ

$$\mathscr{A} \text{ is division } \iff \gamma^i \notin N_{K/F}(K) \quad 1 \leq i < [K:F]$$

- Quaternion algebra $Q = (a, \gamma)_F$ is division $\iff \gamma \notin N_{K/F}(K)$
- Quaternion algebras give fast decodable 2×2 codes.

Iterated Code Construction

- Start with a generalized quaternion algebra $Q(a,\gamma)_F$. We have $\sigma : \sqrt{a} \mapsto \sqrt{a}$.
- Q gives rise to a 2×2 space-time code.
- Iterated construction maps a pair of 2×2 algebraic space-time codewords to a 4×4 MIDO space-time codeword.
- Write σ for the map acting componentwise by

$$\sigma : \begin{bmatrix} c & \gamma\sigma(d) \\ d & \sigma(c) \end{bmatrix} \mapsto \begin{bmatrix} \sigma(c) & \gamma d \\ \sigma(d) & c \end{bmatrix}.$$

Iterated Construction

For θ of K, define $\alpha_\theta : M_2(K) \times M_2(K) \to M_4(K)$

$$\alpha_\theta : (A,B) \mapsto \begin{bmatrix} A & \theta\sigma(B) \\ B & \sigma(A) \end{bmatrix},$$

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$$\alpha_\theta : \left(\begin{bmatrix} c & \gamma\sigma(d) \\ d & \sigma(c), \end{bmatrix}, \begin{bmatrix} e & \gamma\sigma(f) \\ f & \sigma(e), \end{bmatrix} \right) \mapsto \begin{bmatrix} c & \gamma\sigma(d) & \theta\sigma(e) & \theta\gamma f \\ d & \sigma(c) & \theta\sigma(f) & \theta e \\ e & \gamma\sigma(f) & \sigma(c) & \gamma d \\ f & \sigma(e) & \sigma(d) & c \end{bmatrix}. \quad (5)$$

Full Diversity of the Iterated Construction

Lemma (Division Condition)

Let $K = F(\sqrt{a}), \gamma, \theta \in F$ and $Q = (a, \gamma)_F$. Let $\mathscr{A}$ denote the image of α_θ. The algebra $\mathscr{A}$ is division if and only if $\theta \neq z\sigma(z)$ for all $z \in Q$.

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Lemma (Concrete Criteria)

Let $K = F(\sqrt{a}), \gamma, \theta \in F$. We have the following equivalence:

1. $\theta \neq \det \begin{bmatrix} u & \gamma\sigma(v) \\ v & \sigma(u) \end{bmatrix}$, *for any $u, v \in K$ such that $v \in \sqrt{a}F$,* *and*
2. $\theta \neq \gamma (\text{mod } K^{\times 2})$, *where $K^{\times 2}$ denotes the squares in K*

$$\Longleftrightarrow$$

$$\theta \neq z\sigma(z) \text{ for any } z \in Q = (a, \gamma)_F.$$

The nonnorm condition on θ is easily satisfied when F is real, K is imaginary.

Iterated Generalized Alamouti Code

Lemma

Let $\theta = \gamma = -1$, *let* $F = \mathbb{Q}(\sqrt{b})$, $Q = (a, \gamma)_F$ *for* $a < 0, b > 0$.
The complexity of the iterated MIDO code arising from $\alpha_\theta(Q, Q)$ *is* $O(|S|^8)$.

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This code does not have full diversity, however. Have full diversity for when $\theta = -k$, where k is any nonsquare.

Proof.

We show this code is 2-group decodable. Subdivide the basis of the code into two groups $\Gamma_1 \cup \Gamma_2$ so that $AB^* + BA^* = 0$ for all $A \in \Gamma_1$, $B \in \Gamma_2$.

$$\Gamma_1 = \{\alpha_\theta(D,0), \alpha_\theta(0,J)\}, \Gamma_2 = \{\alpha_\theta(J,0), \alpha_\theta(0,D)\},$$

where

$$D = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} \sqrt{b} & 0 \\ 0 & \sqrt{b} \end{bmatrix}, \begin{bmatrix} \sqrt{a} & 0 \\ 0 & -\sqrt{a} \end{bmatrix}, \begin{bmatrix} \sqrt{a}\sqrt{b} & 0 \\ 0 & -\sqrt{a}\sqrt{b} \end{bmatrix} \right\}$$

and

$$J = \left\{ \begin{bmatrix} 0 & \gamma \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \gamma\sqrt{b} \\ \sqrt{b} & 0 \end{bmatrix}, \begin{bmatrix} 0 & -\gamma\sqrt{a} \\ \sqrt{a} & 0 \end{bmatrix}, \begin{bmatrix} 0 & -\gamma\sqrt{a}\sqrt{b} \\ \sqrt{a}\sqrt{b} & 0 \end{bmatrix} \right\}.$$

□

In this case iterated code inherits orthogonality properties of generalized Alamouti code.

Example (Iterated MIDO Alamouti Code)

Pick $F = \mathbb{Q}(\sqrt{b})$, $b > 0$, and $K = F(i)$, with $\sigma : i \mapsto -i$. Then

$$\alpha_\theta : \left(\begin{bmatrix} c & \gamma d^* \\ d & c^* \end{bmatrix}, \begin{bmatrix} e & \gamma f^* \\ f & e^* \end{bmatrix} \right) \mapsto \begin{bmatrix} c & \gamma d^* & \theta e^* & \theta \gamma f \\ d & c^* & \theta f^* & \theta e \\ e & \gamma f^* & c^* & \gamma d \\ f & e^* & d^* & c \end{bmatrix}.$$

Since $c \in F(i)$, $c = c_0 + ic_1$, Now $F = \mathbb{Q}(\sqrt{b})$, and thus

$$c = c_0 + ic_1 = (c_{00} + \sqrt{b}c_{01}) + i(c_{10} + \sqrt{b}c_{11}),$$

with $c_{00}, c_{01}, c_{10}, c_{11} \in \mathbb{Q}$. Alternatively

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$$c = (c_{00} + ic_{10}) + \sqrt{b}(c_{01} + i\sqrt{b}c_{11}),$$

c can encode 2 QAM symbols $c_{00} + ic_{10}$ and $c_{01} + i\sqrt{b}c_{11}$.
The whole MIDO code contains 8 QAM symbols.

Quasi-orthogonal code proposed by Jafarkhani[4] of the Lemma above with $F = \mathbb{Q}$. Note that it transmits only 8 real symbols.

Example (Iterated Alamouti MIDO Code)

Take $F = \mathbb{Q}$ and $K = \mathbb{Q}(i)$, with $\sigma : i \mapsto -i$ the complex conjugation, and $\gamma = -1$.

$$\begin{bmatrix} c & -d^* \\ d & c^* \end{bmatrix}.$$

Now, for $\theta = -1$, we get

$$\alpha_\theta : \left(\begin{bmatrix} c & -d^* \\ d & c^*, \end{bmatrix}, \begin{bmatrix} e & -f^* \\ f & e^*, \end{bmatrix} \right) \mapsto \begin{bmatrix} c & -d^* & -e^* & f \\ d & c^* & -f^* & -e \\ e & -f^* & c^* & -d \\ f & e^* & d^* & c \end{bmatrix}.$$

Iterated Silver MIDO code

Silver Code

The Silver code, discovered in [3], and re-discovered in [6], is given by codewords of the form

$$\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} z_1 & -z_2^* \\ z_2 & z_1^* \end{bmatrix},$$

where

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \frac{1}{\sqrt{7}} \begin{bmatrix} 1+i & -1+2i \\ 1+2i & 1-i \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$$

and $x_1, x_2, x_3, x_4 \in \mathbb{Z}[i]$ are the information symbols.

Alternatively [2], it can be viewed as scaled matrices $\begin{bmatrix} c & -\sigma(d) \\ d & \sigma(c) \end{bmatrix}$, coming from $(-1,-1)_F$, where $F = \mathbb{Q}(\sqrt{-7})$ and $K = F(i)$, with $\sigma : i \mapsto -i$.

Iterated Silver MIDO code

Iterated Silver Code

For $\theta \in F$ we have

$$\alpha_\theta : \left(\begin{bmatrix} c & -\sigma(d) \\ d & \sigma(c) \end{bmatrix}, \begin{bmatrix} e & -\sigma(f) \\ f & \sigma(e), \end{bmatrix} \right) \mapsto \begin{bmatrix} c & -\sigma(d) & \theta\sigma(e) & -\theta f \\ d & \sigma(c) & \theta\sigma(f) & \theta e \\ e & -\sigma(f) & \sigma(c) & -d \\ f & \sigma(e) & \sigma(d) & c \end{bmatrix}.$$

Lemma

The complexity of an iterated Silver MIDO code is at most $O(|S|^{13})$, no matter the choice of θ.

Iterated Silver MIDO code

Lemma (Iterated Silver MIDO Code)

The complexity of the iterated MIDO Silver code with $\theta = -1$ is $O(|S|^{10})$.

It can be verified by direct computation that the R matrix of the iterated MIDO Silver code when $\theta = -1$ has the shape

$$\begin{bmatrix}
t & t & 0 & 0 & 0 & 0 & 0 & 0 & t & t & t & t & t & t & t & t \\
0 & t & 0 & 0 & 0 & 0 & 0 & 0 & t & t & t & t & t & t & t & t \\
0 & 0 & t & t & 0 & 0 & 0 & 0 & t & t & t & t & t & t & t & t \\
0 & 0 & 0 & t & 0 & 0 & 0 & 0 & t & t & t & t & t & t & t & t \\
0 & 0 & 0 & 0 & t & t & 0 & 0 & t & t & t & t & t & t & t & t \\
0 & 0 & 0 & 0 & 0 & t & 0 & 0 & t & t & t & t & t & t & t & t \\
0 & 0 & 0 & 0 & 0 & 0 & t & t & t & t & t & t & t & t & t & t \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & t & t & t & t & t & t & t & t & t \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t & 0 & 0 & 0 & 0 & t & t & t \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t & 0 & 0 & t & 0 & t & t \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t & 0 & t & t & 0 & t \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t & t & t & t & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t
\end{bmatrix}$$

The iterated Silver code is *conditionally* 4-*group decodable*.

Summary of Complexity

Iterated Code	Parameters	Max Complexity	Complexity
Alamouti	$\gamma = 1, \theta = 1$	$O(\lvert S\rvert^{16})$	$O(\lvert S\rvert^{8})$
Silver	$\gamma = 1, \theta \in F$	$O(\lvert S\rvert^{16})$	$O(\lvert S\rvert^{13})$
Silver	$\gamma = 1, \theta = -1$	$O(\lvert S\rvert^{16})$	$O(\lvert S\rvert^{10})$

Performance

Compare performance of Iterated Silver code with $\theta = i$ and complexity $O(|S^{13}|)$.

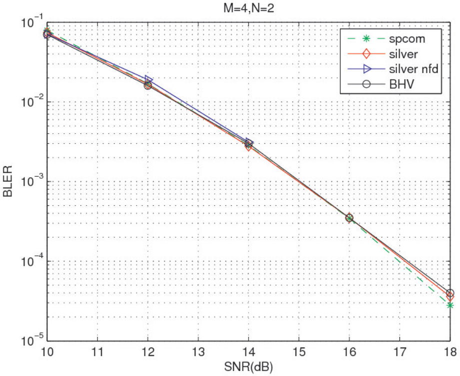

Figure: Comparison among codes with decoding complexity $O(|S|^{12})$ and $O(|S|^{13})$.

Silver $\theta = -1$ vs Crossed Product Algebra Codes

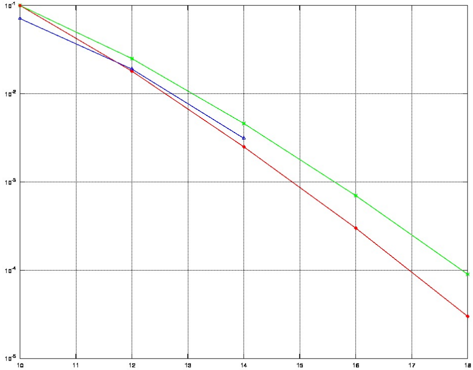

Figure: Comparison among codes with decoding complexity $O(|S|^{10})$.

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