Matrix Codes and Multitone Frequency Shift Keying for Power Line Communications

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Abstract—Single-tone frequency shift keying (FSK) modulation with permutation codes has been found to be useful in addressing the problem of narrowband noise disturbance in power line communications. However, this modulation scheme is restrictive since the number of frequencies used must be at least as large as the number of symbols in the permutation code. In this paper, we propose the use of multitone FSK and binary matrix codes to overcome this restriction. We construct infinite families of efficiently decodable matrix codes with rates and relative distances bounded away from zero, that uses only a logarithmic number of frequencies in the length of the code. Simulation results show that our multitone modulation scheme outperform single-tone modulation schemes.

1. Introduction

Power line communications (PLC) is a technology that enables transmission of data over electric power lines. It was started in the 1910's for voice communication and used in the 1950's in the form of ripple control for load and tariff management in power distribution. With the emergence of the Internet in the 1990's, research into broadband PLC gathered pace as a promising technology for Internet access and local area networking, since the electrical grid infrastructure provides "last mile" connectivity to premises and capillarity within premises. Recently, there has been a renewed interest in high-speed narrowband PLC due to applications in smart grids (see, for instance, [1]).

However, power lines present a difficult communications environment due to the presence of various types of noise, including additive white Gaussian noise, fading, permanent narrowband noise, and impulse noise. Overcoming impulse noise and permanent narrowband noise remains a challenging problem [2]–[4]. Vinck [2] addressed this through the use of a coded modulation scheme, which utilizes permutation codes and multiple frequency shift keying (FSK) as ingredients. There are some inconsistent uses of various notions of FSK in the literature on coded modulation for PLC. Before going further, we clarify this to make clear past and present contributions.

In FSK systems, each symbol is signalled by an element from an alphabet of orthogonal sinusoidal waveforms (*tones*) tuned to different specific frequencies. *Multiple* FSK is a variation of FSK that uses more than two frequencies. FSK schemes can be either *single-tone* or *multitone*.

   (i) *Single-tone* FSK is an FSK scheme where each symbol is signalled by a single tone.
  (ii) *Multitone* FSK is an FSK scheme where each symbol is signalled by a combination of (one or more) tones.

Vinck's scheme is based on single-tone (multiple) FSK, where channel state information is assumed to be unknown to the receiver and a noncoherent demodulator is used. The number of available frequencies must be at least as large as the size of the alphabet over which the code is defined, since the FSK used is single-tone. The use of permutation codes implies that the block length of the code must be equal to the alphabet size. To overcome this restriction, Chee *et al.* [5] extended Vinck's analysis to more general codes and proposed the use of equitable symbol weight codes in conjunction with single-tone FSK for correcting permanent narrowband noise.

Unfortunately, while this more general coded modulation scheme using block codes gives better flexibility and performance, it involves the use of codebooks, which require large storage and do not have efficient decoding algorithms. Coded modulation schemes with low decoding complexity are possible if the size of the code is small enough so that exhaustive search can be performed, or if the codes have sufficient structure such that efficient decoding algorithms can be implemented. Some families of codes with low decoding complexity are given by:

   (i) distance preserving maps from the Hamming space to the permutation space (see [6]–[10]),
  (ii) permutation trellis codes [11],
 (iii) permutation group codes (see [12]), and
 (iv) cosets of Reed-Solomon codes with low symbol weight (see [13], [14]).

The lengths of the families of codes mentioned above are constrained by the number of frequencies, which is at least as large the alphabet size of the code. In particular, the length is at most as large as the number of frequencies. In addition, while the first three families of codes have efficient decoding algorithms, they do not simultaneously achieve positive relative distance and positive rate, with increasing code length.

In this paper, our primary goal is to determine code families with the following (simultaneous) properties:

   (i) positive relative distance,
  (ii) positive rate,
 (iii) have efficient decoding algorithms, and
 (iv) without restriction that the length of the codes is at most the size of the alphabet,

that can be used to combat permanent narrowband noise in PLC. We achieve this by adopting a modification to Vinck's coded modulation scheme that frees up the constraint on code length by alphabet size. Single-tone FSK in Vinck's scheme is replaced by multitone FSK. Luo *et al.* [15] analyzed and

compared the performance of multitone FSK and single-tone FSK schemes in which the signal energy is peaky both in time and frequency. Their results show that both single-tone FSK and multitone FSK, with simple hard-decision decoding, have comparable error performance, and furthermore, both approach the wideband capacity limit at large but finite bandwidths. Verdu [16] also showed that in order to achieve the capacity of a wideband noncoherent fading channel, the signalling must be peaky. Oshinomi *et al.* [17] studied a specific implementation of multitone FSK to demonstrate the spectral efficiency of the model. These results are encouraging with respect to the use of multitone FSK for narrowband PLC. We adopt the use of multitone FSK modulation scheme with the understanding that the energy is concentrated on only a small fraction of the available frequencies.

The use of multitone FSK with only a small number of frequencies at any particular time step corresponds to the use of a bounded weight binary code. We consider a special case of this construction by using a *constant* weight code, in which the weight of every codeword is fixed at some constant, say w. This corresponds to using a combination of exactly w frequencies at any time instance. Each individual codeword in this constant weight code is considered to correspond uniquely to a q-ary symbol. The "outer code", which consists of vectors defined over a q-ary alphabet, is used to determine the block length n, transmitted over n time intervals. This is exactly the definition of a concatenated code [18], [19] with a constant weight code as the inner code and a q-ary code as the outer code. Thus, to determine the set of frequencies to transmit, we use binary matrices as codewords, instead of q-ary vectors. Since these binary matrices are required to have the same number of ones in each column, we are unable to employ general burst-error correcting codes such as array codes [20] or rank-metric codes [21]. Instead, we construct matrix codes meeting our requirements through concatenation. As a result, we establish an infinite family of *efficiently decodable* codes whose rate and relative distance are bounded away from zero, and uses a logarithmic number of frequencies in the length of the code. Simulation results show our multitone modulation scheme outperform single-tone modulation schemes.

The current proposals for communication of narrowband power line channels include the use of orthogonal frequency division multiplexing (OFDM), which also uses multiple tones during transmission [22]. The transmission of the data can also be adaptively modulated depending on the channel characteristics. We do not assume that the channel state information is known to the transmitter or the receiver. Comparisons of the current work with OFDM, and other extensions to include coherent demodulation and adaptive modulation techniques is an interesting avenue for future research.

The rest of this article is organized as follows. In Section 2, we introduce the basic notation and definitions. In Section 4 we look at the worst-case error performance, and the existence of concatenated codes with constant weight inner codes. In this work, we concentrate on using Reed-Solomon codes as outer codes, which provides us with efficient decoding algorithms

for the outer codes. Finally, Section 6 contains simulations of these concatenated codes, assuming hard decision decoding. The concatenated codes are shown to correct more errors than symbol-weight codes of similar rates.

2. Preliminaries and Notations

Denote the ring of integers and the finite field of order q by $\mathbb{Z}$ and $\mathbb{F}_q$, respectively. For a positive integer k, the set $\{1, 2, \ldots, k\}$ is denoted by $[k]$. Given a finite set X, the set of all k-subsets of X is denoted by $\binom{X}{k}$, The *weight* of a vector u is the number of nonzero components in u.

Let Σ be a set of q *symbols*. A q-ary code C of length n over the *alphabet* Σ is a subset of Σ^n. Elements of C are called *codewords*. For $i \in [n]$, denote the ith coordinate of a codeword u by u_i. Endow the space Σ^n with the *Hamming distance* metric. A code $C \subseteq \Sigma^n$ is said to have *distance* d if the (Hamming) distance between any two distinct codewords of C is at least d. A q-ary code of length n and distance d is called an $(n, d)_q$-*code*. An $(n, d)_q$-code whose codewords are all of weight w is called an $(n, d, w)_q$-*constant weight code*, and is denoted by $\mathrm{CW}(n, d, w)_q$.

A. Binary Matrix Codes

Let m, n be positive integers and let $\mathbb{F}_2^{m \times n}$ denote the set of $m \times n$ matrices over $\mathbb{F}_2$. Let $\mathsf{M} \in \mathbb{F}_2^{m \times n}$. We index the rows of M by $[m]$, the columns by $[n]$, and let $\mathsf{M}_{i,j}$ be the (i, j)-th entry of M. We denote the ith row by $\mathsf{M}_{i,*}$ and the jth column by $\mathsf{M}_{*,j}$. A *binary $(m \times n)$-matrix code C* is hence a subset of $\mathbb{F}_2^{m \times n}$. The code C is said to have *constant column weight* w if each column of a matrix in C has weight w.

B. Concatenated Codes

Let B be an $(n, d_B)_q$-code over Σ and A be an $(m, d_A)_2$-code with $|A| \geq q$. Let $\psi : \Sigma \to A$ be any injective mapping and we write $\psi(\sigma)$ as a binary column vector of length m. Then the *concatenated code* $A \circ B$ defined by *inner code* A, *outer code* B and *mapping* ψ is the following set of $m \times n$ matrices over $\mathbb{F}_2$:

$$A \circ B = \{\mathsf{M} : \mathsf{M}_{*,j} = \psi(u_j), j \in [n], u \in B\}.$$

The *inner distance* of $A \circ B$ is d_A and its *outer distance* is d_B. The size of $A \circ B$ is $|B|$. Note that elements of $A \circ B$ are binary $m \times n$ matrices, so $A \circ B$ is a binary matrix code. If in addition, A is a constant weight code of weight w, then $A \circ B$ has constant column weight w and $A \circ B$ is called an $(m \times n, d_A, d_B, w)$-*concatenated constant column weight code*, and is denoted by $\mathrm{CCW}(m \times n, d_A, d_B, w)$.

3. Coded Modulation with Multitone FSK

We modify Vinck's coded modulation scheme to use a binary matrix code in conjunction with multitone FSK, where each symbol is signalled by a combination of w different tones from an alphabet of m tones. We call such a multitone FSK an $\binom{m}{w}$-FSK. An $\binom{m}{1}$-FSK corresponds to the single-tone FSK used by Vinck.

Consider a binary $(m \times n)$-matrix code C with constant column weight w. Each codeword in C corresponds to a

message. We use an $\binom{m}{w}$-FSK with alphabet $\{f_1, f_2, \ldots, f_m\}$. To transmit a message corresponding to $\mathsf{M} \in C$, we transmit n symbols, each of which is signalled by a combination of w tones, $\{f_i : i \in [m], \mathsf{M}_{i,j} = 1\}$, $j \in [n]$, over n discrete time steps. We can therefore think of each codeword in C as having rows indexed by tones and columns indexed by time steps. The rate of this code C is

$$R(C) = \frac{\log |C|}{n \log \binom{m}{w}}.$$

We remark that this definition of the rate captures the size of the "space" when we use w frequencies in n time instances. This differs from the definition in [10], [11] where the rate is defined as the number of bits transmitted per channel use.

Example 3.1. The message corresponding to the codeword

$$\mathsf{M} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

is transmitted via the sets of tones $\{f_3, f_4\}$, $\{f_2, f_4\}$, $\{f_2, f_3\}$, $\{f_1, f_4\}$, and $\{f_1, f_3\}$ over five discrete time steps.

Assuming a hard-decision threshold detector, the received signal (which may contain errors caused by noise) is demodulated to an output $\mathsf{N} \in \mathbb{F}_2^{m \times n}$. The burst errors that arise from the different types of noises in the PLC channel (see [23, pp. 222–223]) have the following effects on the detector output.

(i) A narrowband noise introduces a tone at all time instances of the transmitted signals. If $e \in [m]$ and e *narrowband noise errors* occur, then there is a set $\Gamma \in \binom{[m]}{e}$ of e rows, such that $\mathsf{N}_{i,j} = 1$ for $i \in \Gamma$, $j \in [n]$.

(ii) Impulse noise results in the entire set of tones being received at a certain time instance. If $e \in [n]$ and e *impulse noise errors* occur, then there is a set $\Pi \in \binom{[n]}{e}$ of e columns such that $\mathsf{N}_{i,j} = 1$ for $i \in [m]$, $j \in \Pi$.

(iii) A channel fade event erases a particular tone. If $e \in [m]$, and e *fades* occur then there is a set $\Gamma \in \binom{[m]}{e}$ of e rows such that $N_{i,j} = 0$ for all $j \in [n]$.

(iv) Background noise flips the value of the bit at a particular tone and time instance. If e *background noise* occurs then there exists a set $\Omega \in \binom{[n] \times [m]}{e}$ such that $N_{i,j} = M_{i,j} + 1$, for all $(i, j) \in \Omega$.

More simply, a narrowband noise turns an entire row of N to ones, an impulse noise turns an entire column of N to ones, a channel fade event turns an entire row of N to zeros, and a background noise flips an entry of N.

Example 3.2. Continuing Example 3.1, if one narrowband noise error occur at frequency 1 and one impulse noise occur at time instance 2, the resulting demodulated matrix is

$$\mathsf{N} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

The next section describes a decoding algorithm for our coded modulation scheme.

Algorithm 1: decoder for concatenated codes

Input: detector output $\mathsf{N} \in \mathbb{F}_2^{m \times n}$
Output: $\mathsf{N}' \in A \circ B$

1 **for** $j \in [n]$ **do**
2   **if** $\mathsf{N}_{i,j} = 1$ *for all* $i \in [m]$ **then**
3      $\mathsf{v}_j \leftarrow ?$
4   **else**
5   decode $\mathsf{N}_{*,j}$ to $\mathsf{c}_j \in A$
6      $\mathsf{v}_j \leftarrow \psi^{-1}(\mathsf{c}_j)$
7   **end**
8 **end**
9 decode v to $\mathsf{u} \in B$
10 **for** $j \in [n]$ **do**
11    $\mathsf{N}'_{*,j} \leftarrow \psi(\mathsf{u}_j)$
12 **end**
13 **return** N'

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4. DECODING

We follow the usual method of decoding concatenated codes by decoding the inner code first, followed by decoding the outer code. Below, we present the sufficient conditions under which correct decoding can be performed.

Let $A \circ B$ be an $(m \times n, d_A, d_B, w)$-concatenated constant column weight code. Let Σ be the alphabet for B and $\psi : \Sigma \rightarrow A$ be the injective map defining $A \circ B$. For the code B we use a bounded distance decoder that corrects both errors and erasures, and for the code A we use a minimum distance decoder which also corrects both errors and erasures. Suppose the detector output is $\mathsf{N} \in \mathbb{F}_2^{m \times n}$. We decode N to $\mathsf{N}' \in A \circ B$ in two steps. First, we decode N to a codeword $\mathsf{v} \in (\Sigma \cup \{?\})^n$, where $?$ is the erasure symbol. For $j \in [n]$, if the column $\mathsf{N}_{*,j}$ is an all-one vector, we set the v_j to be $?$. Otherwise, we decode the column $\mathsf{N}_{*,j}$ to a codeword in A, and using ψ, convert this codeword to $\mathsf{v}_j \in \Sigma$. Next, we decode v to a codeword $\mathsf{u} \in B$. Using ψ again, we represent the codeword u as a matrix $\mathsf{N}' \in A \circ B$. See Algorithm 1 for details.

The conditions for correct decoding are given in the following proposition. For simplicity, consider the case where only narrowband noise and impulse noise are present. The sufficient conditions can be readily extended to the case when background noise and fading are also present.

Proposition 4.1. Let $A \circ B$ be an $(m \times n, d_A, d_B, w)$-concatenated constant column weight code. $A \circ B$ is able to correct e_{NB} narrowband noise errors and e_{IM} impulse noise errors if $2e_{\mathsf{NB}} < d_A$, $e_{\mathsf{NB}} + w < m$, and $e_{\mathsf{IM}} < d_B$.

The inequality $e_{\mathsf{NB}} + w < m$ captures the situation where a column of all ones is *not* introduced by the presence of narrowband noise errors.

5. CODE CONSTRUCTION

We are after concatenated codes with relative outer and/or inner distances bounded away from zero (to guarantee good error-correcting capabilities implied by Proposition 4.1), have efficient decoding algorithms for decoding of outer and inner codes, and have rates bounded away from zero. To achieve this,

we use Reed-Solomon codes as outer codes. Denote a q-ary Reed-Solomon code of length n, dimension k and minimum distance d by $\mathrm{RS}[n, k, d]_q$. The following theorem gives a general construction of an efficiently decodable concatenated constant column weight code.

Theorem 5.1. Let $n+1$ be a prime power with $n+1 \le |A|$. Let the outer code B be an $\mathrm{RS}[n, n - d_B + 1, d_B]_{n+1}$ and inner code A be a $\mathrm{CW}(m, d_A, w)_2$. Then $A \circ B$ is a $\mathrm{CCW}(m \times n, d_A, d_B, w)$ of rate $(1 - (d_B - 1)/n)(\log n / \log \binom{m}{w})$ and decoding complexity $O(n^2) + O(n|A|)$.

We specialize Theorem 5.1 in two ways to give two families of asymptotically good codes.

A. Codes from Block Designs

Our first specialization of Theorem 5.1 comes from application of combinatorial designs. An $(m, w, 1)$-BIBD (*balanced incomplete block design*) is a pair $(X, \mathcal{B})$ such that $|X| = m$ and $\mathcal{B}$ is a set of w-subsets of X, called *blocks*, with the property that every 2-subset of X is contained in exactly one block. Wilson [24] showed that for every fixed w, there exists an $(m, w, 1)$-BIBD for all sufficiently large m satisfying the congruences $m(m - 1) \equiv 0 \bmod w(w - 1)$ and $m - 1 \equiv 0 \bmod w - 1$. The blocks of an $(m, w, 1)$-BIBD form the supports of a $\mathrm{CW}(m, 2(w - 1), w)_2$ of size $m(m - 1)/(w(w - 1))$.

Corollary 5.1 (Block Design Construction). Fix $w \ge 2$ and let $0 < \delta_B < 1$. Then there exists a $\mathrm{CCW}(m \times n, d_A, d_B, w)$ C_m, for all sufficiently large m satisfying $m(m - 1) \equiv 0 \bmod w(w - 1)$ and $m - 1 \equiv 0 \bmod w - 1$, where
(i) $n = \Theta(m^2)$, (ii) $d_A = 2(w - 1)$, (iii) $d_B = \lceil \delta_B n \rceil$.
Furthermore, this code family has the property that $\lim_{m \to \infty} R(C_m) \ge 2(1 - \delta_B)/w$.

Proof: For sufficiently large m satisfying $m(m - 1) \equiv 0 \bmod w(w - 1)$ and $m - 1 \equiv 0 \bmod w - 1$, take a $\mathrm{CW}(m, 2(w - 1), w)_2$ of size $m(m - 1)/(w(w - 1))$ as inner code A. Let $n + 1$ be a prime power such that $m(m-1)/(2w(w-1)) \le n+1 \le m(m-1)/(w(w-1))$, and let $d_n = \lceil \delta_B n \rceil$. Take an $\mathrm{RS}[n, n - d_B + 1, d_B]_{n+1}$ as outer code B. Then $A \circ B$ is the desired $\mathrm{CCW}(m \times n, d_A, d_B, w)$.
Conditions (i)–(iii) are immediate. The asymptotic rate of the code family can be verified as follows:

$$\lim_{m \to \infty} \left(1 - \frac{d_B - 1}{n}\right) \frac{\log n}{\log \binom{m}{w}}$$
$$\ge (1 - \delta_B) \lim_{m \to \infty} \frac{\log(m^2/(2w(w - 1)))}{\log m^w} \ge 2 \frac{(1 - \delta_B)}{w}.$$

B. Codes via Gilbert-Varshamov Construction

Our second specialization is based on the Gilbert-Varshamov construction. Levenshteĭn [25] showed that when applied to the space of constant weight vectors, the Gilbert-Varshamov construction gives, for fixed positive $\delta, \kappa < 1$, a $\mathrm{CW}(m, \delta m, \kappa m)_2$ of size at least $2^{m(H(\kappa) - s(\delta, \kappa))}$, where

$$s(\delta, \kappa) = \max_{0 \le \sigma \le \delta/2} \kappa H(\sigma/\kappa) + (1 - \kappa) H(\sigma/(1 - \kappa)).$$

Corollary 5.2 (Gilbert-Varshamov Construction). Fix $0 < \delta_A < \kappa < 1/2$, $0 < \delta_B < 1$. Then for m sufficiently large, there exists a $\mathrm{CCW}(m \times n, d_A, d_B, w)$ C_m such that
(i) $n = \Theta\left(2^{m(H(\kappa) - s(\delta_A, \kappa))}\right)$, (ii) $d_A = \lceil \delta_A m \rceil$,
(iii) $d_B = \lceil \delta_B n \rceil$, (iv) $w = \lceil \kappa m \rceil$.
Furthermore, this code family has the property that $\lim_{m \to \infty} R(C_m) \ge (1 - \delta_B)(1 - s(\delta_A, \kappa)/H(\kappa))$.

6. SIMULATIONS

We simulate the performance of concatenated constant column weight codes in the presence of narrowband noise. The setup is as follows. Let m be the number of tones used, n be the number of discrete time steps taken to transmit a symbol, and $0 < p < 1$. We simulate a PLC channel with the following independent error characteristics:

(i) for each $i \in [m]$, a narrowband noise error occurs at tone i with probability p, and is present for a duration of δn, where δ is chosen uniformly at random from the set [10],

(ii) for each $j \in [n]$, an impulse noise error occurs at time instance j with probability 0.05,

(iii) for each $i \in [m]$, a channel fade event occurs at frequency i with probability 0.05, and

(iv) for each $i \in [m]$ and $j \in [n]$, a background noise occurs at frequency i and time instance j with probability 0.05.

We choose 10^5 random codewords M from each code under comparison to transmit through the simulated PLC channel. At the receiver, we decode the detector output N to the codeword N' using Algorithm 1. The number of symbols in error when transmitting a codeword is then $|\{j \in [n] : \mathsf{M}_{*,j} \ne \mathsf{N}'_{*,j}\}|$, and the *error rate* is the fraction of time steps in error.

We compare the performance of concatenated constant column weight codes with low symbol weight cosets of RS codes of similar rates. Such symbol weight codes were studied by Versfeld *et al.* [13], [14]. A code C over alphabet Σ has bounded symbol weight r if all symbols in Σ appears at most r times in all codewords in C. Versfeld *et al.* [13], [14] showed that there exists a coset of $\mathrm{RS}[n, k, n - k + 1]_q$ with bounded symbol weight k. Denote such a code by $\mathrm{RSC}[n, k, n-k+1]_q$.

Consider an $\mathrm{RSC}[n, k, n-k+1]_q$. Identify the elements in $\mathbb{F}_q$ with elements in $[q]$ and for codeword u we transmit the matrix $\mathsf{M} \in \mathbb{F}_2^{q \times n}$, where

$$\mathsf{M}_{i,j} = \begin{cases} 1 & \text{if } \mathsf{u}_j = i \\ 0 & \text{otherwise.} \end{cases}$$

At the receiver, we decode the detector output N to a codeword u' using the algorithm described in [13], [14]. The number of symbols in error is then $d(\mathsf{u}, \mathsf{u}')$ and the *error rate* is the ratio of the total number of symbols in error to the total number of symbols transmitted.

We compare concatenated constant column weight codes and low weight cosets of Reed-Solomon codes of similar rates. The parameters of the codes under comparison are given in Table I and the results of the simulations are given in Fig. 1. Observe that concatenated constant column weight codes achieve significantly lower error rates as compared to the low weight cosets of Reed-Solomon codes.

TABLE I

COMPARING LOW SYMBOL WEIGHT COSETS OF REED-SOLOMON CODES AND CONCATENATED CONSTANT COLUMN WEIGHT CODES

Code	Length	Number of tones	Rate	Remarks
$RSC[15, 8, 8]_{16}$	15	16	$8/15 \approx 0.533$	coset of Reed-Solomon code
$CW(9, 4, 4)_2 \circ RS[15, 14, 2]_{16}$	15	9	$(14 \log 16)/(15 \log 126) \approx 0.535$	concatenated constant column weight code
$RSC[15, 5, 11]_{16}$	15	16	$1/3 \approx 0.333$	coset of Reed-Solomon code
$CW(13, 6, 5)_2 \circ RS[15, 14, 2]_{16}$	15	13	$(14 \log 16)/(15 \log 1287) \approx 0.361$	concatenated constant column weight code
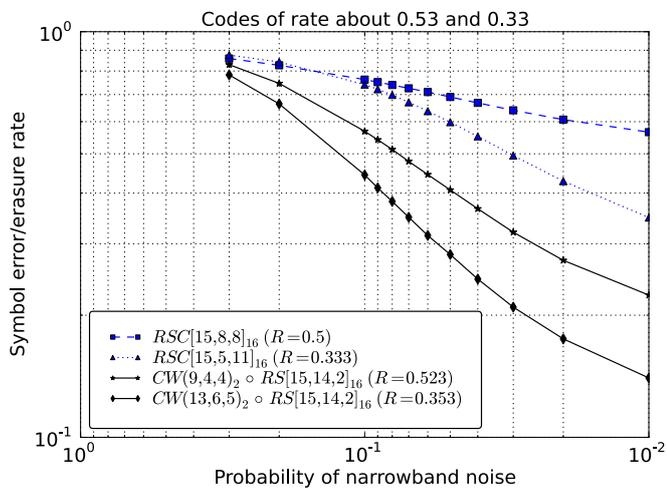
Fig. 1. Comparing concatenated and Reed Solomon codes of similar rates

7. CONCLUSION

We introduce a coded modulation scheme for PLC based on multitone FSK and concatenated constant column weight codes. We construct families of efficiently decodable concatenated constant column weight codes for this scheme with rate and relative distances bounded away from zero. Simulations show our scheme achieves lower error rates as compared to Vinck's scheme based on low symbol weight cosets of Reed-Solomon codes.

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