# Preimages for Step-Reduced SHA-2

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NTU, 25 Nov 2009

A merged version with Aoki, Sasaki and Wang will appear in ASIACRYPT 2009

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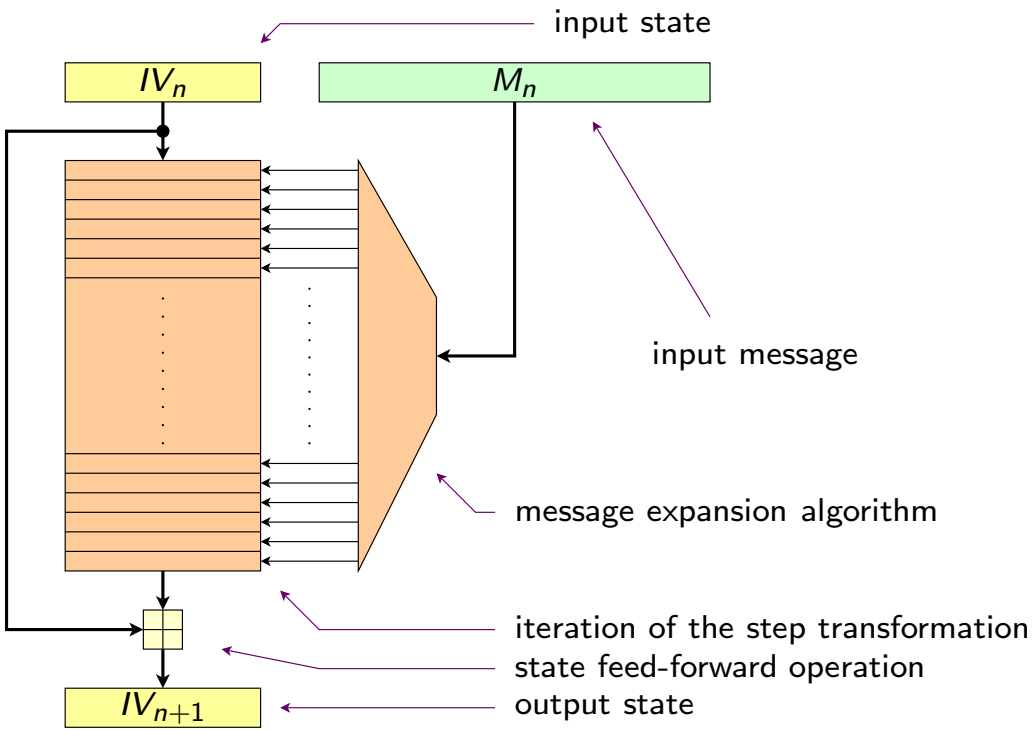
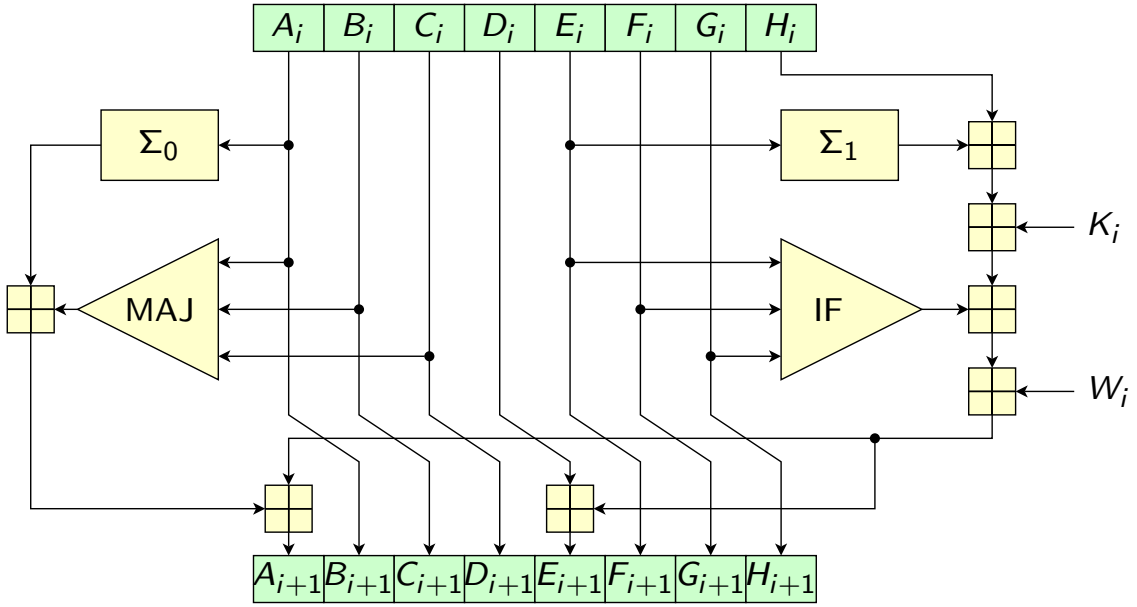
# SHA-2 in General



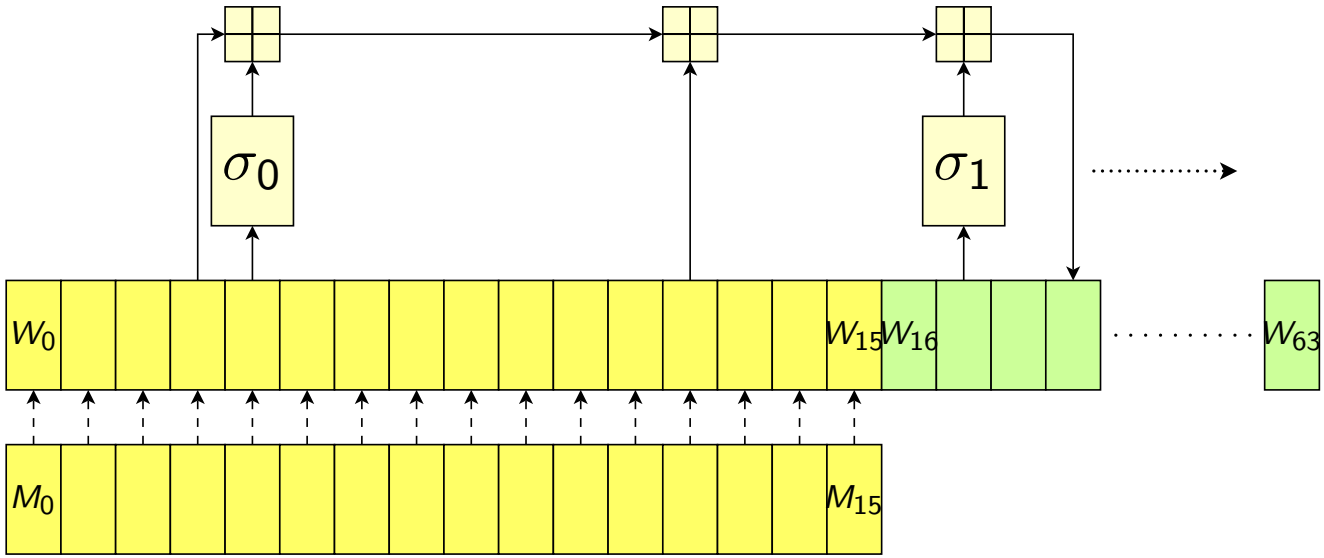

- Step Function: update internal chaining
- Message Expansion: expand 16 message words to 64/80

# SHA-2 Step Function




$$\mathrm{MAJ}(A,B,C) = (A \wedge B) \vee (A \wedge C) \vee (B \wedge C) \ ,$$
$$\mathrm{IF}(E,F,G) = (E \wedge F) \vee (\neg E \wedge G) \ ,$$
$$\Sigma_0(x) = (x \ggg 2) \oplus (x \ggg 13) \oplus (x \ggg 22) \ ,$$
$$\Sigma_1(x) = (x \ggg 6) \oplus (x \ggg 11) \oplus (x \ggg 25) \ .$$

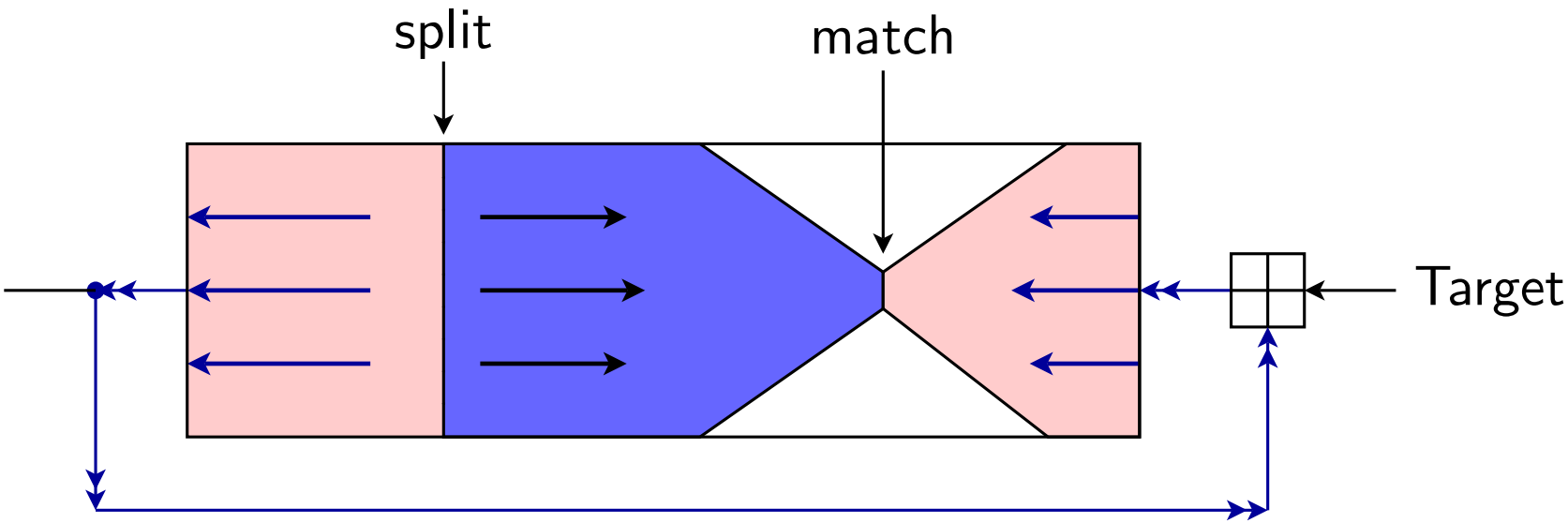
# SHA-2 Message Expansion




$$W_i = \begin{cases} M_i & \text{for } 0 \leq i < 16 \ , \\ \sigma_1(W_{i-2}) + W_{i-7} + \sigma_0(W_{i-15}) + W_{i-16} & \text{for } 16 \leq i < 64 \ . \end{cases}$$

Note: any consecutive 16 determine all message words.

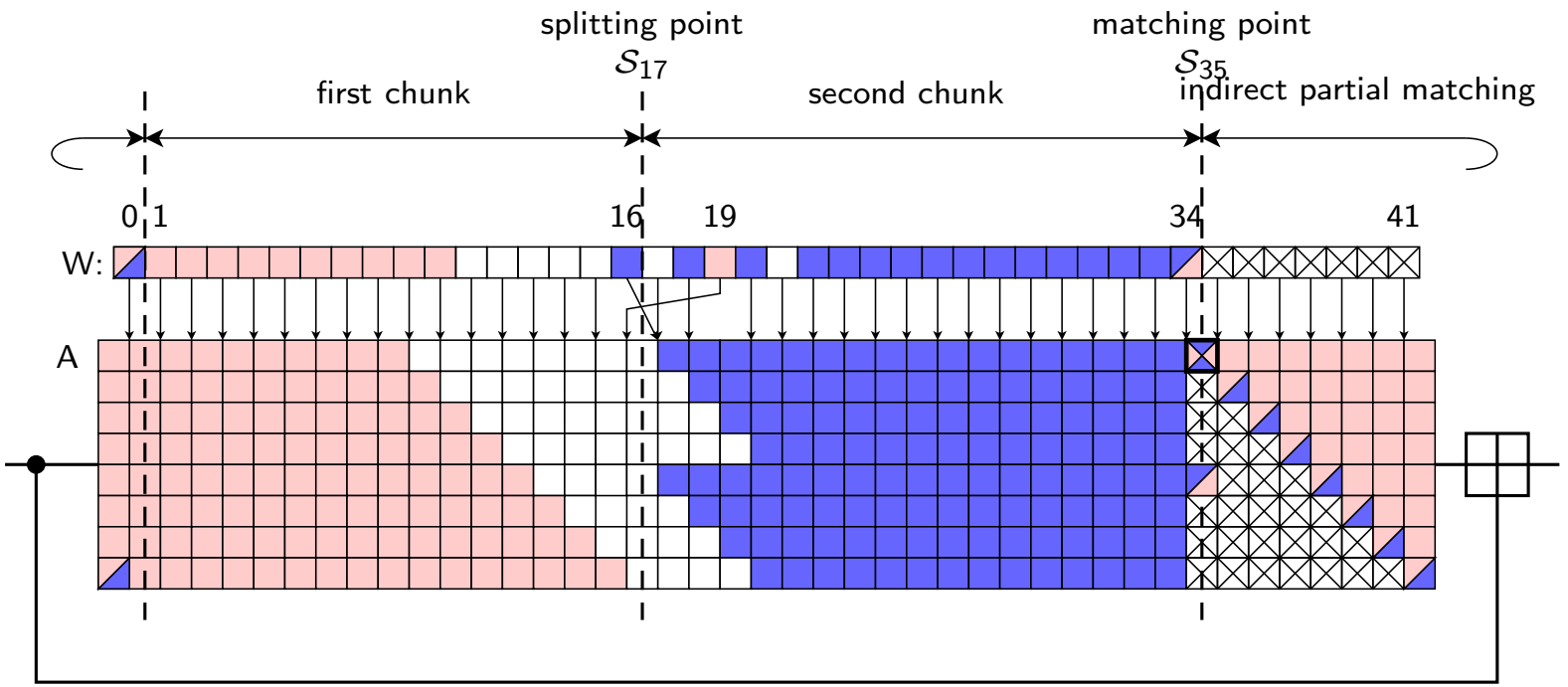
# Preimage Attack - in general



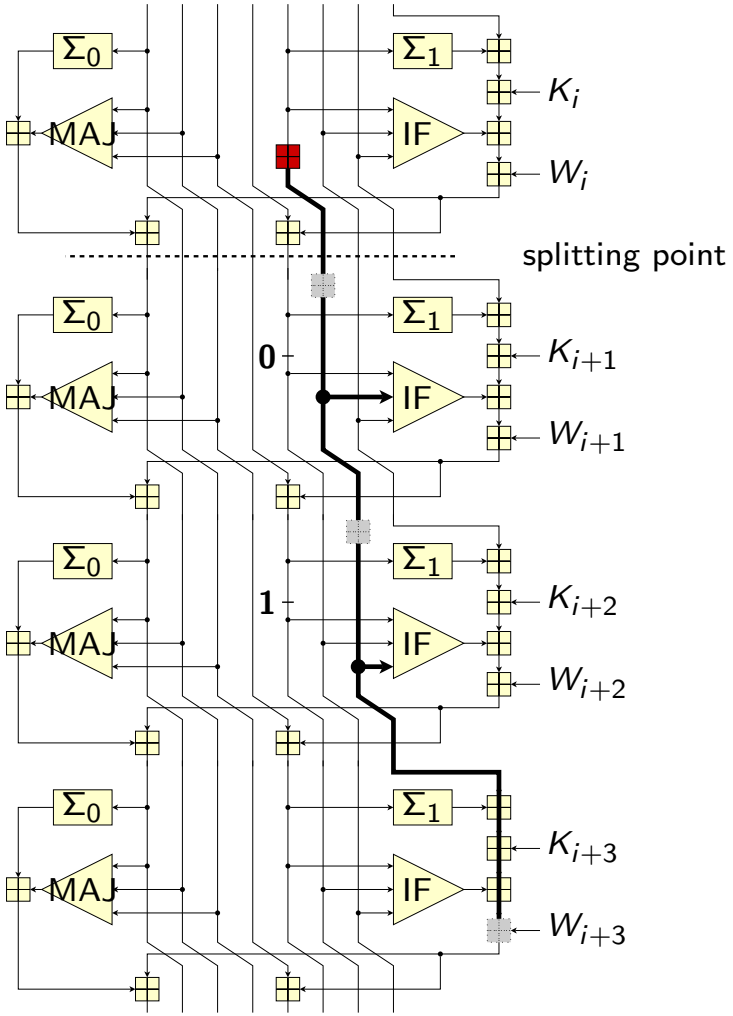

Find pseudo-preimage in $2^l$, then preimage in $2^{\frac{n+l}{2}+1}$

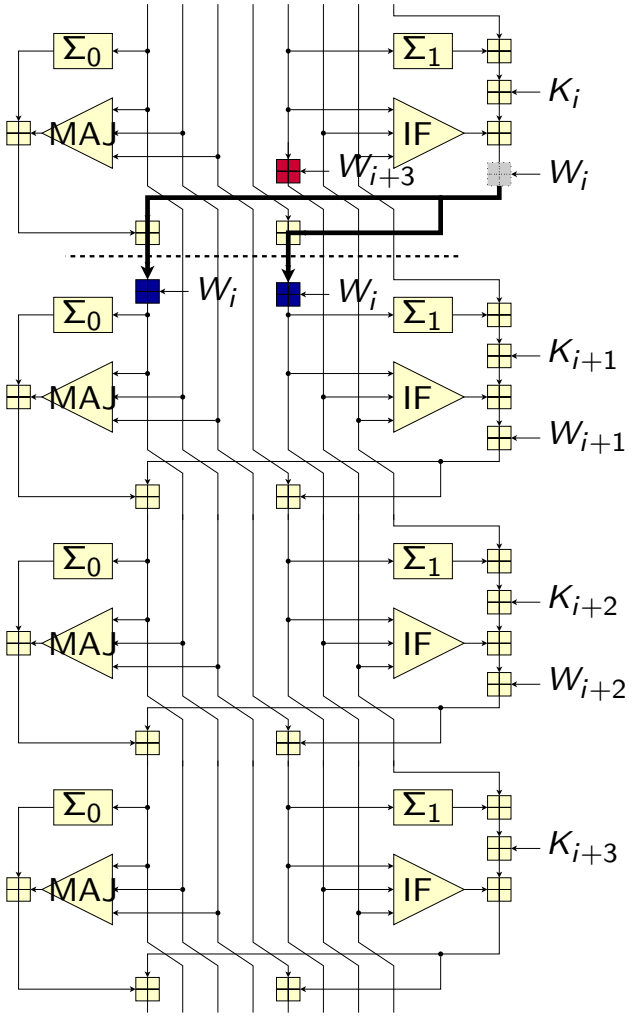
# Result on SHA-2

- $W_{11}, \ldots, W_{26}$ as a basis to generate all message words.
- Neutral words: $W_{16}$ and $W_{19}$



# Message Stealing



# Result on SHA-2

- $W_{11}, \ldots, W_{26}$ as a basis to generate all message words.
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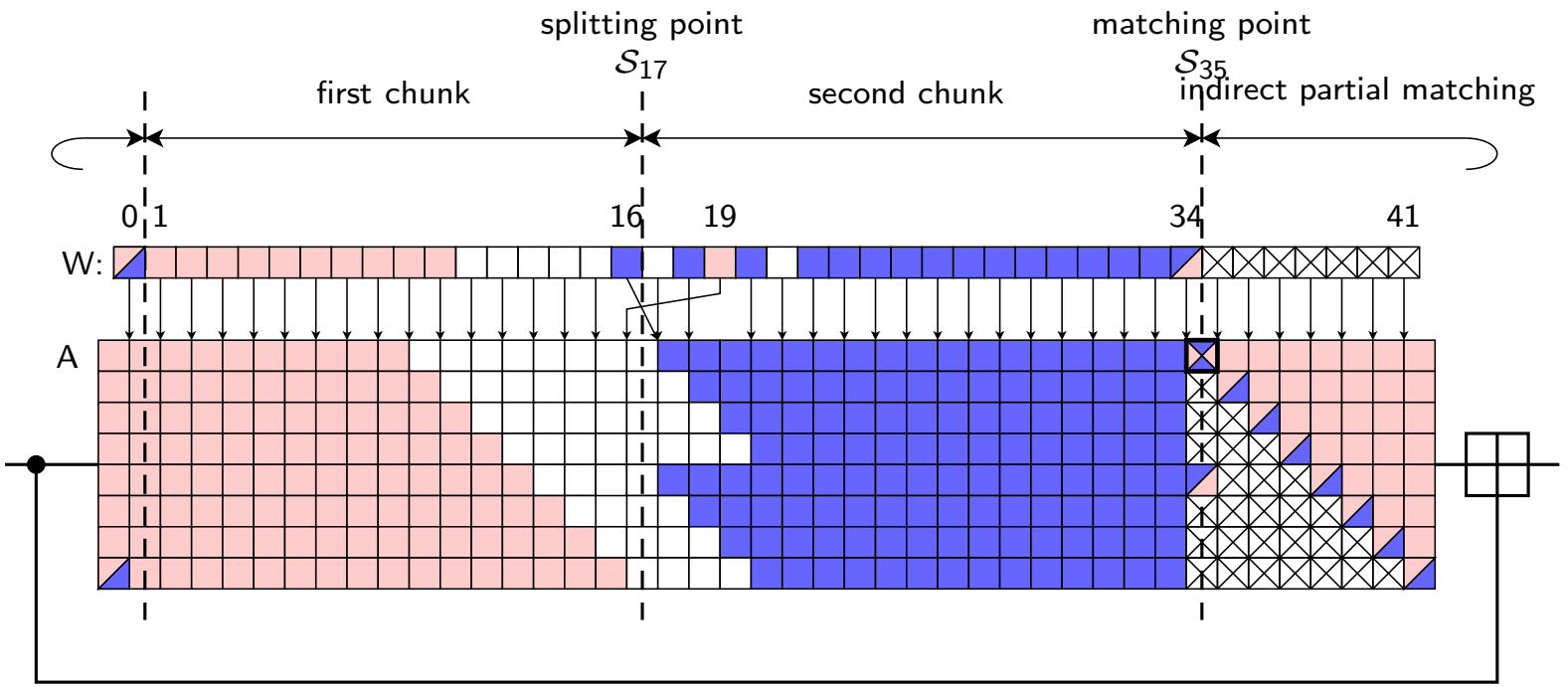

# Message Compensation - First Chunk

$$
\begin{aligned}
W_{10} &= W_{26} - \sigma_1(W_{24}) - W_{19} - \sigma_0(W_{11}) \ , \\
W_9 &= W_{25} - \sigma_1(W_{23}) - W_{18} - \sigma_0(W_{10}) \ , \\
W_8 &= W_{24} - \sigma_1(W_{22}) - W_{17} - \sigma_0(W_9) \ , \\
W_7 &= W_{23} - \sigma_1(W_{21}) - W_{16} - \sigma_0(W_8) \ , \\
W_6 &= W_{22} - \sigma_1(W_{20}) - W_{15} - \sigma_0(W_7) \ , \\
W_5 &= W_{21} - \sigma_1(W_{19}) - W_{14} - \sigma_0(W_6) \ , \\
W_4 &= W_{20} - \sigma_1(W_{18}) - W_{13} - \sigma_0(W_5) \ , \\
W_3 &= W_{19} - \sigma_1(W_{17}) - W_{12} - \sigma_0(W_4) \ , \\
W_2 &= W_{18} - \sigma_1(W_{16}) - W_{11} - \sigma_0(W_3) \ , \\
W_1 &= W_{17} - \sigma_1(W_{15}) - W_{10} - \sigma_0(W_2) \ , \\
W_0 &= W_{16} - \sigma_1(W_{14}) - W_9 - \sigma_0(W_1) \ .
\end{aligned}
$$

# Message Compensation - First Chunk

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\begin{aligned}
W_{10} &= W_{26} - \sigma_1(W_{24}) - W_{19} - \sigma_0(W_{11}) \ , \\
W_9 &= W_{25} - \sigma_1(W_{23}) - W_{18} - \sigma_0(W_{10}) \ , \\
W_8 &= W_{24} - \sigma_1(W_{22}) - W_{17} - \sigma_0(W_9) \ , \\
W_7 &= W_{23} - \sigma_1(W_{21}) - W_{16} - \sigma_0(W_8) \ , \\
W_6 &= W_{22} - \sigma_1(W_{20}) - W_{15} - \sigma_0(W_7) \ , \\
W_5 &= W_{21} - \sigma_1(W_{19}) - W_{14} - \sigma_0(W_6) \ , \\
W_4 &= W_{20} - \sigma_1(W_{18}) - W_{13} - \sigma_0(W_5) \ , \\
W_3 &= W_{19} - \sigma_1(W_{17}) - W_{12} - \sigma_0(W_4) \ , \\
W_2 &= W_{18} - \sigma_1(W_{16}) - W_{11} - \sigma_0(W_3) \ , \\
W_1 &= W_{17} - \sigma_1(W_{15}) - W_{10} - \sigma_0(W_2) \ , \\
W_0 &= W_{16} - \sigma_1(W_{14}) - W_9 - \sigma_0(W_1) \ .
\end{aligned}
$$

# Message Compensation - First Chunk

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W_{10} &= W_{26} - \sigma_1(W_{24}) - W_{19} - \sigma_0(W_{11}) \ , \\
W_9 &= W_{25} - \sigma_1(W_{23}) - W_{18} - \sigma_0(W_{10}) \ , \\
W_8 &= W_{24} - \sigma_1(W_{22}) - W_{17} - \sigma_0(W_9) \ , \\
W_7 &= W_{23} - \sigma_1(W_{21}) - W_{16} - \sigma_0(W_8) \ , \\
W_6 &= W_{22} - \sigma_1(W_{20}) - W_{15} - \sigma_0(W_7) \ , \\
W_5 &= W_{21} - \sigma_1(W_{19}) - W_{14} - \sigma_0(W_6) \ , \\
W_4 &= W_{20} - \sigma_1(W_{18}) - W_{13} - \sigma_0(W_5) \ , \\
W_3 &= W_{19} - \sigma_1(W_{17}) - W_{12} - \sigma_0(W_4) \ , \\
W_2 &= W_{18} - \sigma_1(W_{16}) - W_{11} - \sigma_0(W_3) \ , \\
W_1 &= W_{17} - \sigma_1(W_{15}) - W_{10} - \sigma_0(W_2) \ , \\
W_0 &= W_{16} - \sigma_1(W_{14}) - W_9 - \sigma_0(W_1) \ .
\end{aligned}
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# Message Compensation - First Chunk

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\begin{aligned}
W_{10} &= W_{26} - \sigma_1(W_{24}) - W_{19} - \sigma_0(W_{11}) \ , \\
W_9 &= W_{25} - \sigma_1(W_{23}) - W_{18} - \sigma_0(W_{10}) \ , \\
W_8 &= W_{24} - \sigma_1(W_{22}) - W_{17} - \sigma_0(W_9) \ , \\
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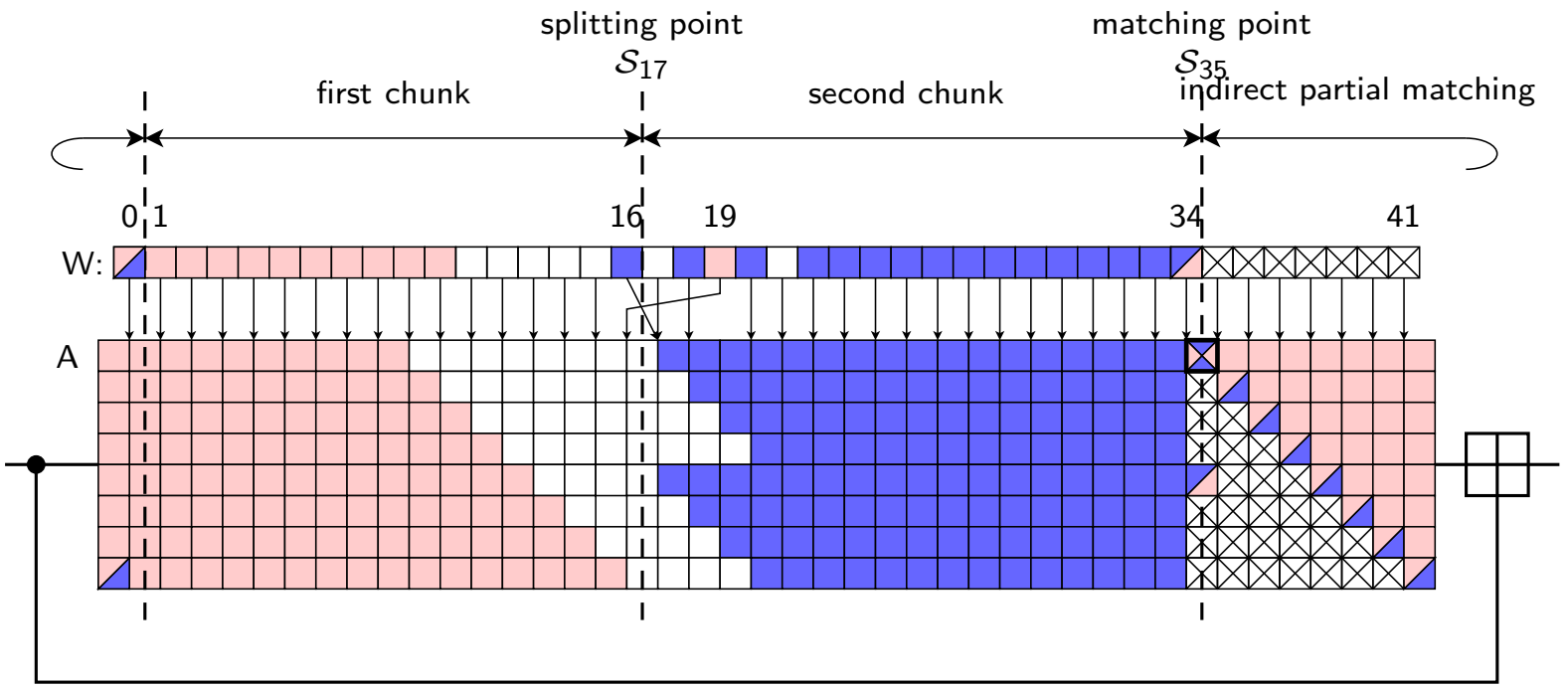
W_5 &= W_{21} - \sigma_1(W_{19}) - W_{14} - \sigma_0(W_6) \ , \\
W_4 &= W_{20} - \sigma_1(W_{18}) - W_{13} - \sigma_0(W_5) \ , \\
W_3 &= W_{19} - \sigma_1(W_{17}) - W_{12} - \sigma_0(W_4) \ , \\
W_2 &= W_{18} - \sigma_1(W_{16}) - W_{11} - \sigma_0(W_3) \ , \\
W_1 &= W_{17} - \sigma_1(W_{15}) - W_{10} - \sigma_0(W_2) \ , \\
W_0 &= W_{16} - \sigma_1(W_{14}) - W_9 - \sigma_0(W_1) \ .
\end{aligned}
$$

# Result on SHA-2

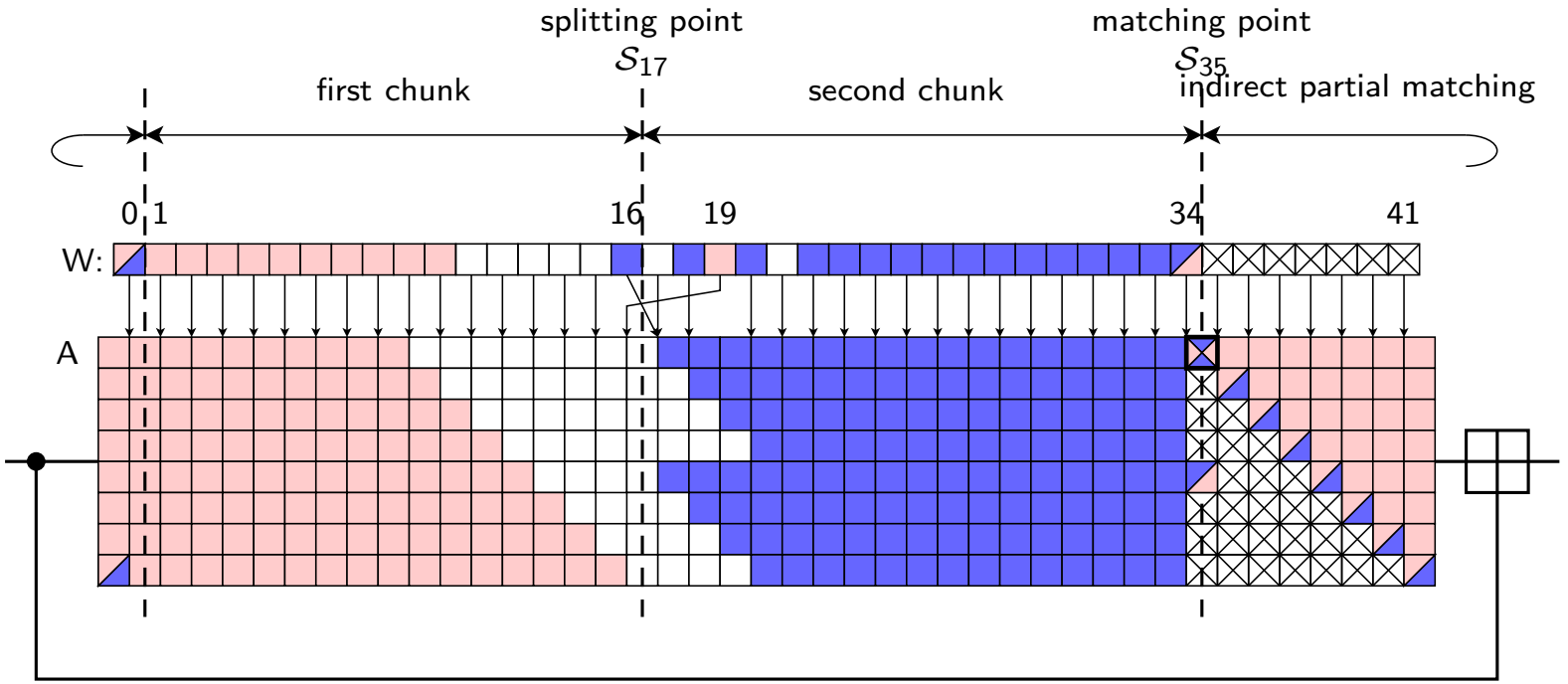
- $W_{11}, \ldots, W_{26}$ as a basis to generate all message words.
- Neutral words: $W_{16}$ and $W_{19}$



# Message Compensation - Second Chunk

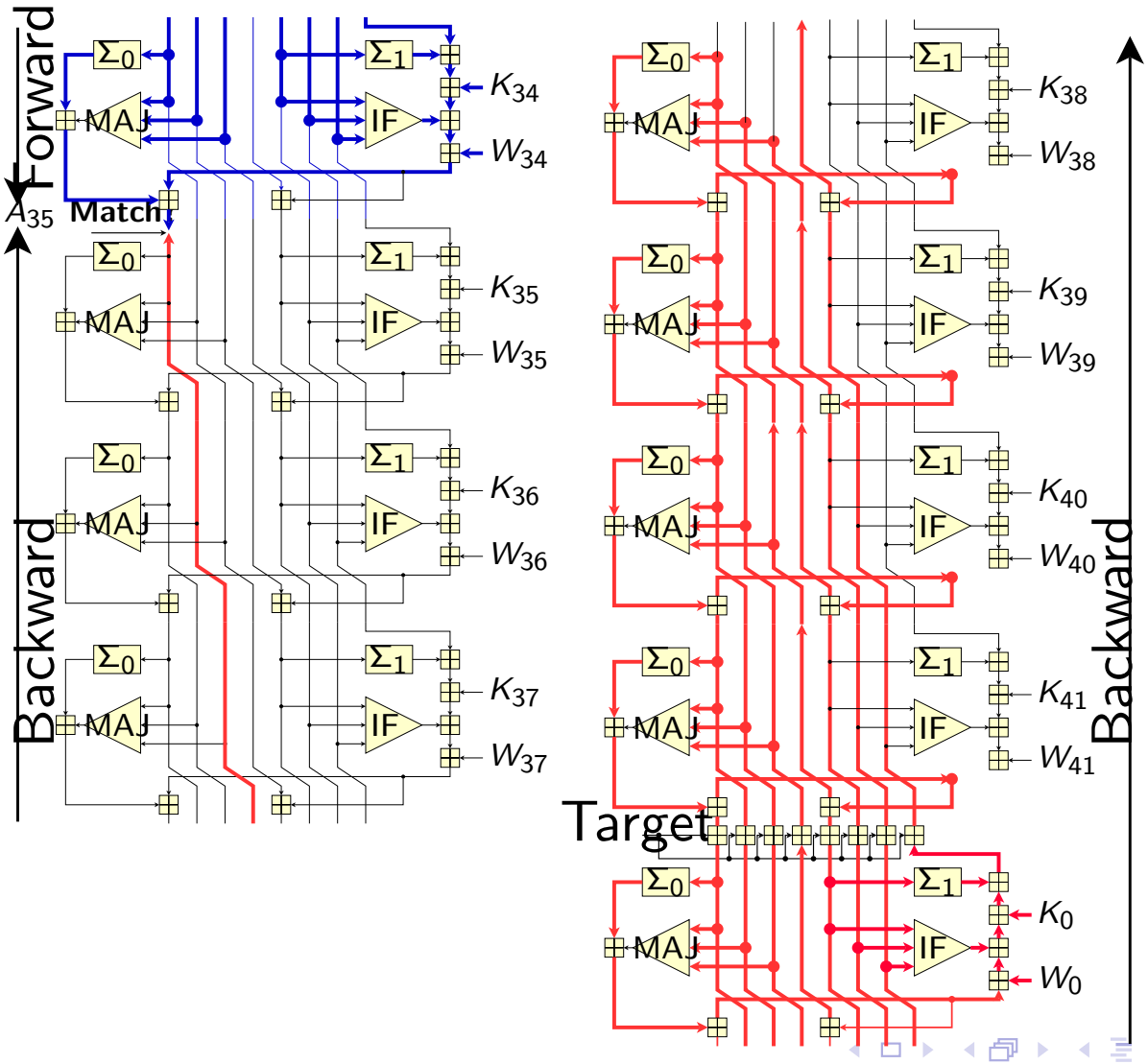
$$
\begin{aligned}
W_{27} &= \sigma_1(W_{25}) + W_{20} + \sigma_0(W_{12}) + W_{11} \ , \\
W_{28} &= \sigma_1(W_{26}) + W_{21} + \sigma_0(W_{13}) + W_{12} \ , \\
W_{29} &= \sigma_1(W_{27}) + W_{22} + \sigma_0(W_{14}) + W_{13} \ , \\
W_{30} &= \sigma_1(W_{28}) + W_{23} + \sigma_0(W_{15}) + W_{14} \ , \\
W_{31} &= \sigma_1(W_{29}) + W_{24} + \sigma_0(W_{16}) + W_{15} \ , \\
W_{32} &= \sigma_1(W_{30}) + W_{25} + \sigma_0(W_{17}) + W_{16} \ , \\
W_{33} &= \sigma_1(W_{31}) + W_{26} + \sigma_0(W_{18}) + W_{17} \ , \\
W_{34} &= \sigma_1(W_{32}) + W_{27} + \sigma_0(W_{19}) + W_{18} \ .
\end{aligned}
$$

# Result on SHA-2




$$W_0 = W_{16} - \sigma_1(W_{14}) - W_9 - \sigma_0(W_1)$$
$$W_{34} = \sigma_1(W_{32}) + W_{27} + \sigma_0(W_{19}) + W_{18}$$

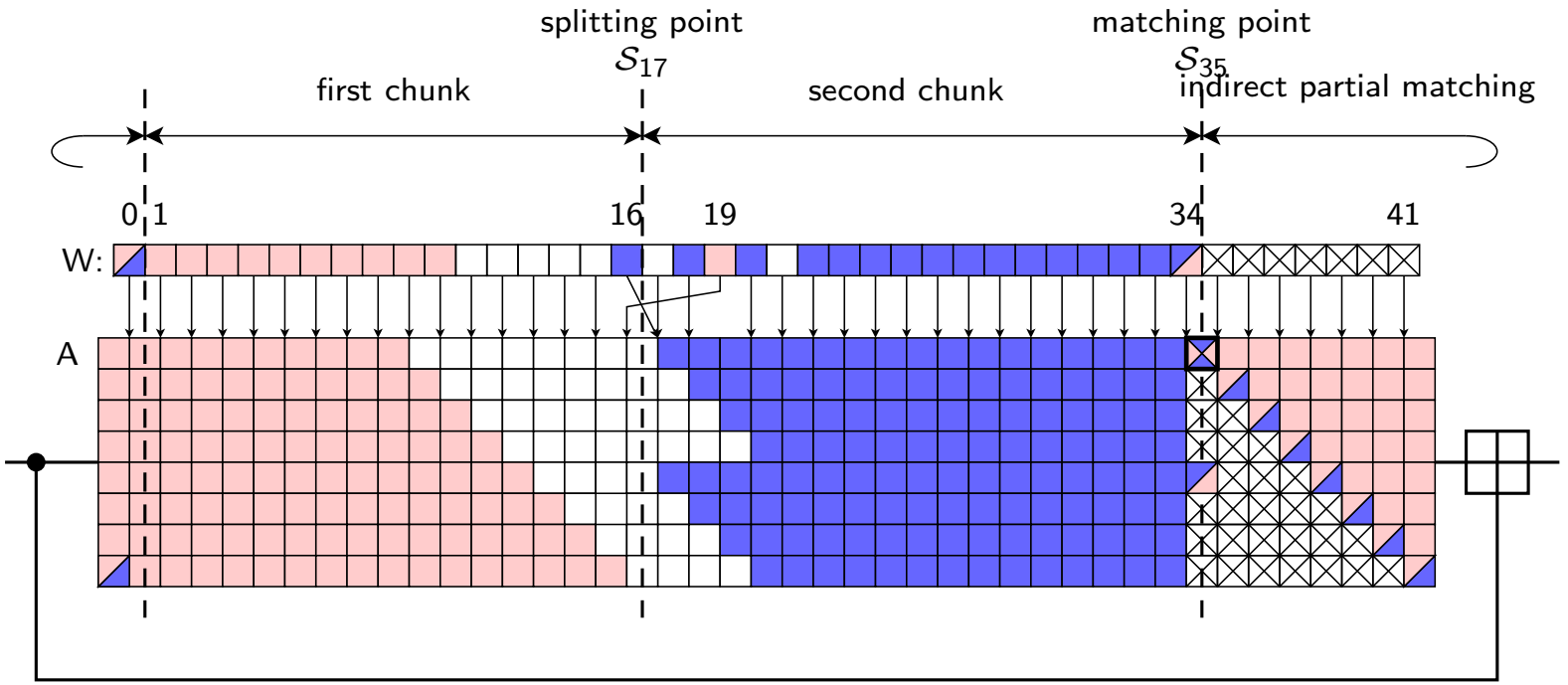
# Extended Partial Matching



# Extended Partial Matching

$$\begin{aligned} & \psi(W_{16}) + \sigma_0(W_{19}) && = \mu(W_{19}) - W_{16} \\ \Longleftrightarrow \quad & \psi(W_{16}) + W_{16} && = \mu(W_{19}) - \sigma_0(W_{19}) \end{aligned}$$

# Result on SHA-2



# Conclusions

- We find preimages for 42 out of 64 (66%) step-reduced SHA-256 with complexity $2^{251.7}$ and memory requirement of order $2^{12}$ bits
- The same attack applies to SHA-512 with complexity $2^{502.3}$ and memory requirement of order $2^{22}$

# END

THANK YOU!
QUESTIONS?